Risk Factor Detection with Methods from Explainable ML

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Quant Insights Conference

22 March 2023

Hochschule für Wirtschaft und Recht Berlin
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Overview

Explainable ML

- Machine Learning (ML) techniques are now commonly used in finance applications to process large amounts of data.
- Ongoing challenges are missing transparency and missing interpretability: How do predictions and forecasts relate to the inputs?
- This will become more important with ongoing regulatory changes, e.g. (EC, 2021; EBA, 2021).
- Here: First results from a research project on explainable ML funded by IFAF for the next two years.

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Explainable ML

- Application in mind is **stress testing**.
- Classical setting: Factor model with **observable factors** (e.g. geographic regions, industries).
- Giving **latent factors** an interpretation extends range of stress scenarios.
- Concrete case: Use **Principal Component Analysis (PCA)** to determine latent factors from class factors and give them an interpretation.
- Idea goes back to work recent work on stress testing, (Packham and Woebbeking, 2019, 2023).

Overview

Linear factor model

- **Linear factor model**: Express vector of asset returns $(r_1, \dots, r_p)$ as

$$r_i = \alpha_i + \beta_{i1} F_1 + \beta_{i2} F_2 + \cdots + \beta_{id} F_d + \varepsilon_i, \qquad i = 1, \dots, p,$$

  where

  - $F_1, \dots, F_d$: return in **common factors**,
  - $\beta_{i1}, \beta_{i2}, \dots, \beta_{id}$: **factor coefficients** or **factor weights**,[1]
  - α_i: constant,
  - ε_i: **residual** or **idiosyncratic component**.

- Common assumption: residuals are uncorrelated.
- Number of factors small compared to number of securities, $d \ll p$.

[1] Sometimes called loadings. We will use the term "loadings" in a slightly different context.

Linear factor model

- Factors $F_1, \dots F_d$ **observable**, e.g. index returns of geographic regions and industries (MSCI GICS).
- Dependence structure of large portfolios expressed via covariances of common factors.
- Decompose $p \times p$ covariance matrix of returns $(r_1, \dots, r_p)$ into

$$\Sigma \approx B \, \Omega \, B^T,$$

  where
  - B: $p \times d$ matrix of factor coefficients,
  - Ω: $d \times d$ covariance matrix of common factors, and
  - we ignore the variances of the residuals.
- Examples of factor models in credit risk management: Moody's KMV, CreditMetrics (by RiskMetrics), see e.g. Bluhm *et al.* (2003).

Classical stress testing

- For "classical" stress testing method, see e.g. (Kupiec, 1998; Dowd, 2002; Packham and Woebbeking, 2019).
- Separate factors into **"core"** and **"peripheral" factors**.
- $\boldsymbol{F}_s$: $j < d$ **core factor returns** that are **stressed directly**.
- Remaining $d - j$ peripheral factor returns $\boldsymbol{F}_u$ indirectly affected by stress scenario.
- Under normal distribution assumption, optimal estimator of $\boldsymbol{F}_u | \boldsymbol{F}_s$[2]:

$$\mathbb{E}(\boldsymbol{F}_u | \boldsymbol{F}_s) = \Sigma_{us} \Sigma_{ss}^{-1} \boldsymbol{F}_s,$$

  where Σ_{us} and Σ_{ss} denote covariance and variance matrices of $\boldsymbol{F}_u$ and $\boldsymbol{F}_s$.
- See (Bonti *et al.*, 2006) for more advanced stress testing method.

[2] For simplicity, we assume the factor returns have expectation zero

Stress testing with latent factors

- Goal here is to expand the universe of risk factors by **aggregating** existing factors into new factors.
- Examples: Global risk factor, European risk factor, cyclical industries, etc.
- Idea:
  - Use PCA on **observable factors** to determine aggregated (latent) factors.
  - Give these factors an interpretation.

Overview

Principal Component Analysis

- In $\mathbb{R}^n$, **PCA** refers to a particular rotation of the axes, driven by random variables or data.
- Key idea is to align random variables / data such that
  - first dimension captures maximal variance,
  - second dimension is orthogonal and captures second-most variance,
  - etc.
- **Principal components (PCs)** are the **eigenvectors** of covariance / correlation matrix.
- **Eigenvalues** express **amount of variance** captured by each PC.

Principal Component Analysis

- See James *et al.* (2013), Section 10.2, for the following.
- Given $n \times d$ data set $\mathbf{X}$ that is **standardised**.
- **First principal component**: find **scores**

$$z_{i1} = \phi_{11} x_{i1} + \phi_{21} x_{i2} + \cdots + \phi_{d1} x_{id}, \quad i = 1, \ldots, n,$$

  that have largest sample variance, subject to constraint $\sum_{j=1}^{p} \phi_{j1}^2 = 1$.
- In other words, **first PC vector**[3] solves optimisation problem

$$\max_{\phi_{11},\ldots,\phi_{p1}} \left\{ \frac{1}{n} \sum_{i=1}^{n} \underbrace{\left(\sum_{j=1}^{p} \phi_{j1} x_{ij} \right)^2}_{= z_{i1}^2} \right\} \quad \text{subject to} \quad \sum_{j=1}^{p} \phi_{j1}^2 = 1.$$

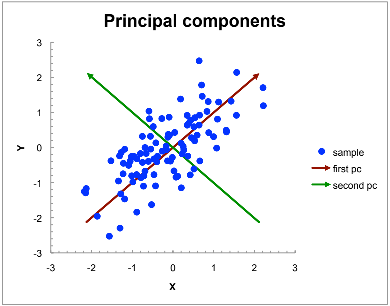
- Second (and higher) PCs: linear combination of data uncorrelated with first PC(s) and with largest variance (subject to constraint).

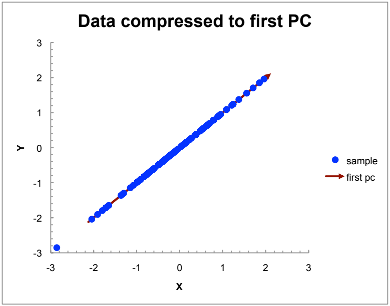
[3] Called loading vector in (James *et al.*, 2013).

Principal Component Analysis

- Compact notation (recall that $\mathbf{X}$ is standardised):
$$\mathbf{Z} = \mathbf{\Phi}^T \mathbf{X}$$
- PCs can be viewed as factors, giving factor model
$$\mathbf{X} = \mathbf{\Phi} \mathbf{Z}.$$
- $\mathbf{\Phi}$ are the eigenvectors of correlation matrix of $\mathbf{X}$.

- Example:


Principal components
3
2
1
Y 0
-1
-2
-3
-3 -2 -1 0 1 2 3
X
sample
first pc
second pc


Data compressed to first PC
3
2
1
Y 0
-1
-2
-3
-3 -2 -1 0 1 2 3
X
sample
first pc

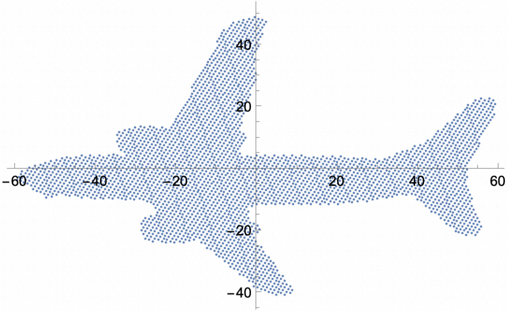
- Example from Mathematica:

In[264]:=

`shape = Position[ImageData@``, 1, {2}];`

```
ListPlot[PrincipalComponents[N@shape]]
```

Out[265]=



Overview

Overview

Data

- Geographical factors: 16 regions and countries represented by MSCI indices
- Industry factors: 11 MSCI Global Industry Classification Standard (GICS) sector indices
- Daily data, split into Jan 1999-Dec 2019 (train) and Jan 2020-Feb 2023 (test)
- Data from Refinitiv Eikon
- Data split into **six groups**:
  - Europe (developed)
  - Asia-Pacific (developed)
  - N. America
  - Emerging Markets (Europe, M. East, Africa, Asia, Latin Am.)
  - Cyclical industries
  - Defensive industries

Overview

Loadings

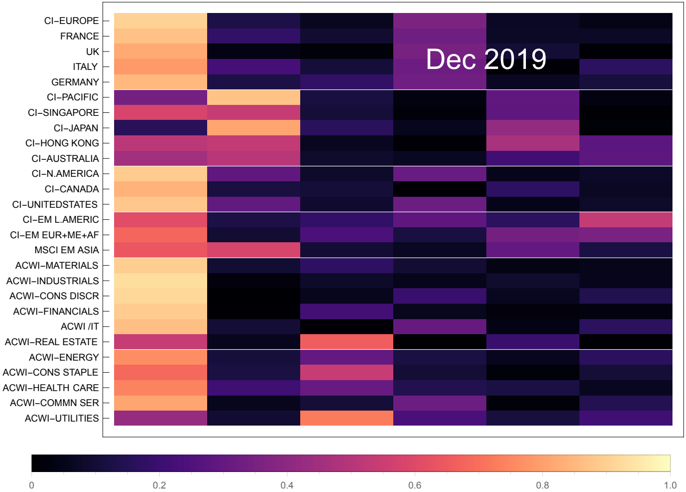
- Giving PC's an interpretation: correlation between data and scores (= projection of data to PC).
- Assume that data standardised.
- Using that the PC's are uncorrelated and have variances λ_i, $i = 1, \dots, d$:

$$\text{Corr}(x_{\cdot j}, z_{\cdot i}) = \frac{\text{Cov}(x_{\cdot j}, z_{\cdot i})}{\sqrt{\lambda_i}} = \frac{\mathbb{E}[\phi_{ji} z_{\cdot i} z_{\cdot i}]}{\sqrt{\lambda_i}} = \phi_{ji}\sqrt{\lambda_i}.$$

- In words: correlation of data and scores are just PCs scaled with PC standard deviation ("importance" of PC).
- In-line with $\approx 50\%$ of the literature, we shall call these **loadings**.

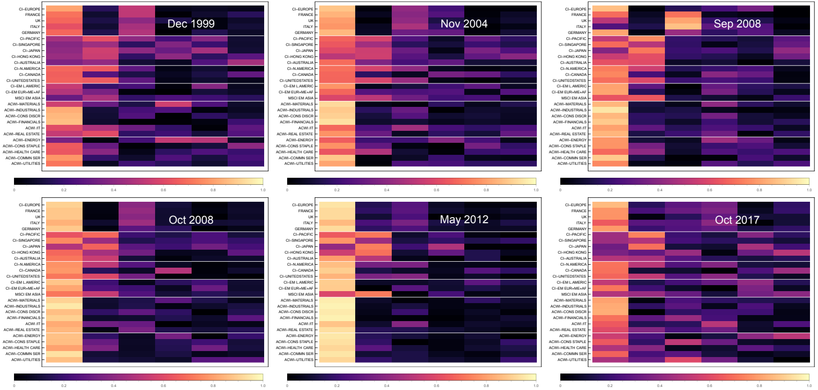
Loadings

- (Absolute) loadings, i.e., correlations of factor returns with first PCs:



Loadings

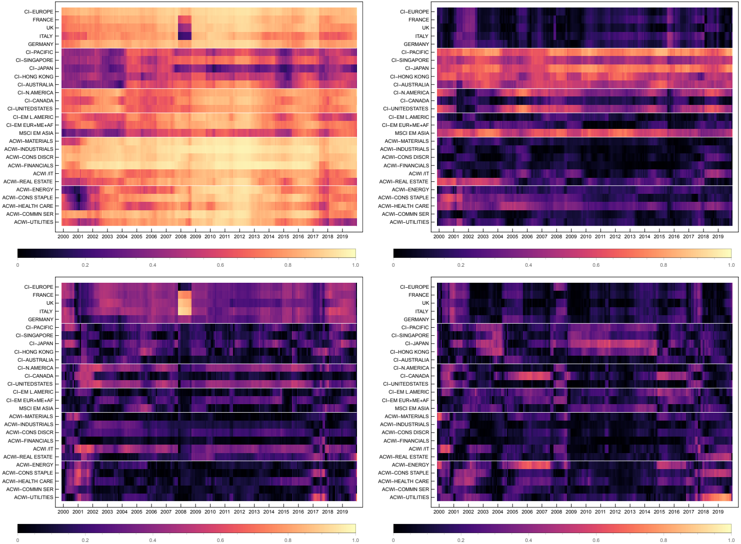
- PCA at the end of each month on a rolling window of 250 days.
- A few more loadings plots:



- And a movie: Open Movie, Download movie

Loadings

- Loadings of PCs through time (top: PC1, PC2; bottom: PC3, PC4):



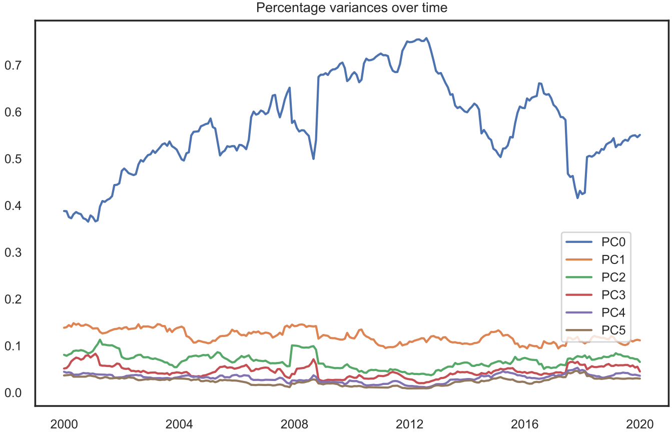
Overview

Interpretation of PCs

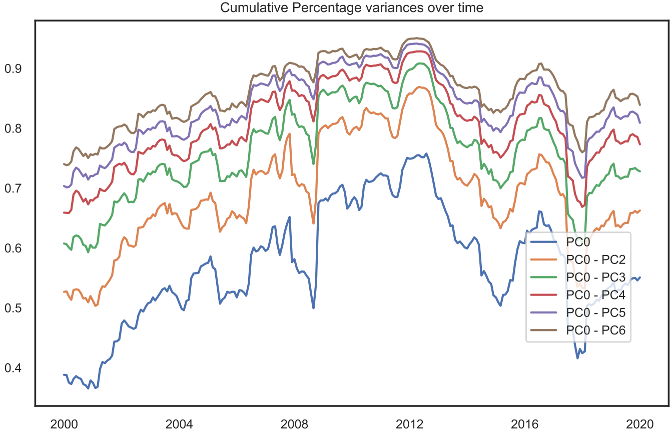
- Two questions:
  - How many PCs are relevant?
  - Which geographic region or industry group does PC explain?
- Literature: (Fenn *et al.*, 2011)

Overview

How many PCs are relevant?

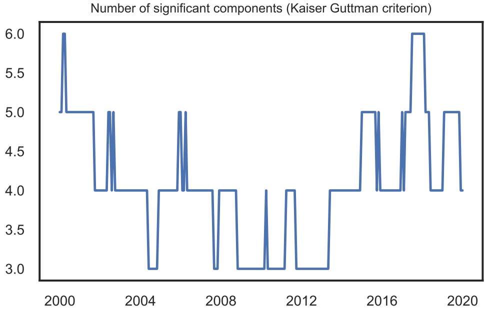


How many PCs are relevant?



Kaiser-Guttman criterion

- The **Kaiser-Guttman criterion** measures the number of significant PCs.
- The i-th PC is significant if its (normalised) eigenvalue λ_i is greater than $1/d$, where d is the number of eigenvalues.
- Idea: A PC that satisfies this criterion accounts for more than a fraction $1/d$ of the variance.
- See e.g. (Fenn *et al.*, 2011; Guttman, 1954).



Overview

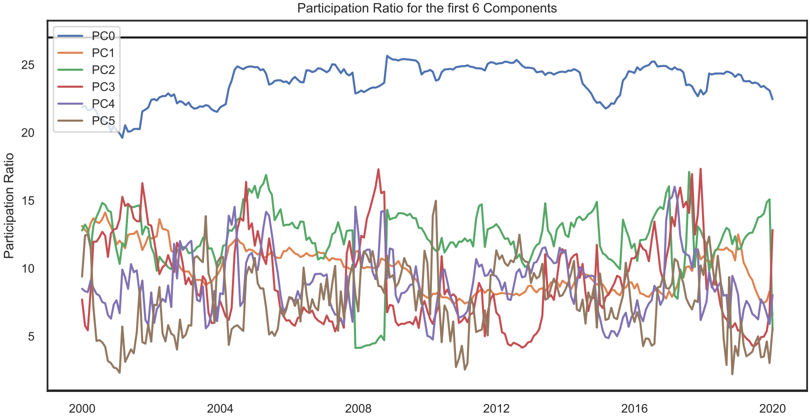
What drives changes in PCs?

- **Inverse participation ratio (IPR)** of i-th PC: (Fenn *et al.*, 2011; Guhr *et al.*, 1998):

$$I_k = \sum_{j=1}^{d} (\phi_{ji})^4.$$

- IPR measures number of assets participating in a PC:
  - eigenvector with equal contributions $\phi_{ji} = 1/\sqrt{d}$ has $I_k = 1/d$;
  - eigenvector with single contribution $\phi_{ji} = 1$ (others zero) has $I_k = 1$.
- **Participation ratio (PR)**: $1/I_k$
- Large PR: Many assets contribute

Participation Ratio



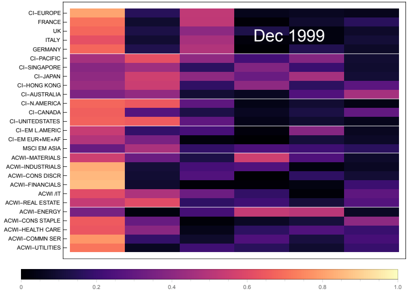
Overview

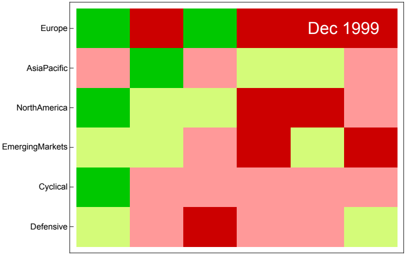
Which geographic region or industry group does PC explain?

- Six groups:
  - Europe (developed)
  - Asia-Pacific (developed)
  - N. America
  - Emerging Markets
  - Cyclical industries
  - Defensive industries
- For a given PC and its PR, define the **PR group** as the group of **size PR** of indices with **highest loadings**.
- Group explained / not explained by a particular PC:

Strong In: *All* indices in a group are in the PR group.	Strong Out: *No* indices in a group are in the PR group.
Weak In: *More than half* of indices in a group are in the PR group.	Weak Out: *Half or less* of indices in a group are in the PR group.

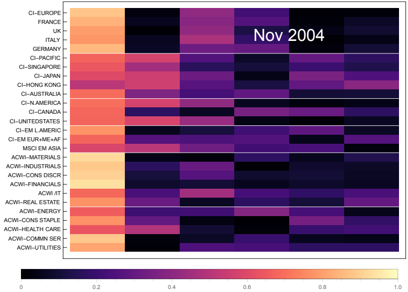
PC interpretation



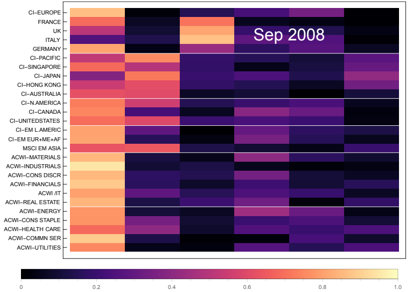


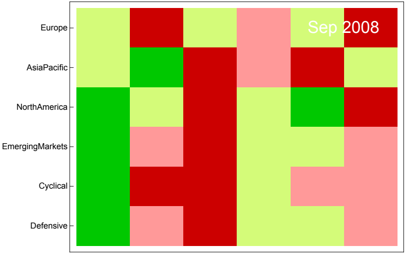
- First PC: global ex-AsiaPacific
- Second PC: AsiaPacific, EM, NA
- Third PC: Europe, NA

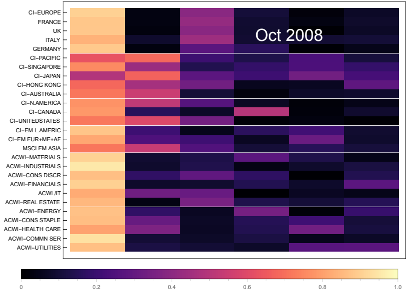
PC interpretation

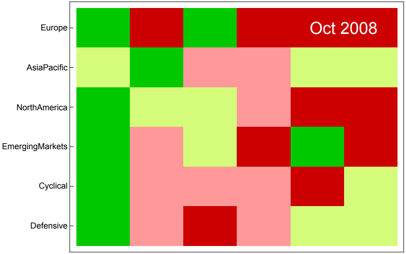


PC interpretation

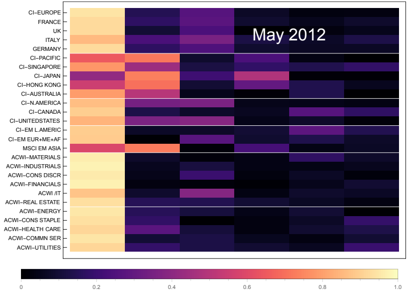


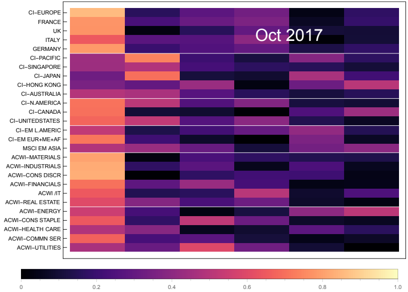


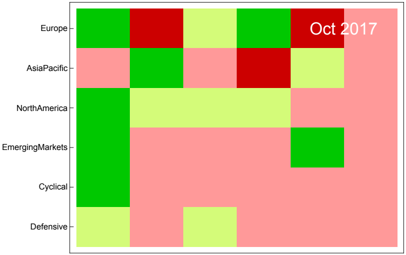




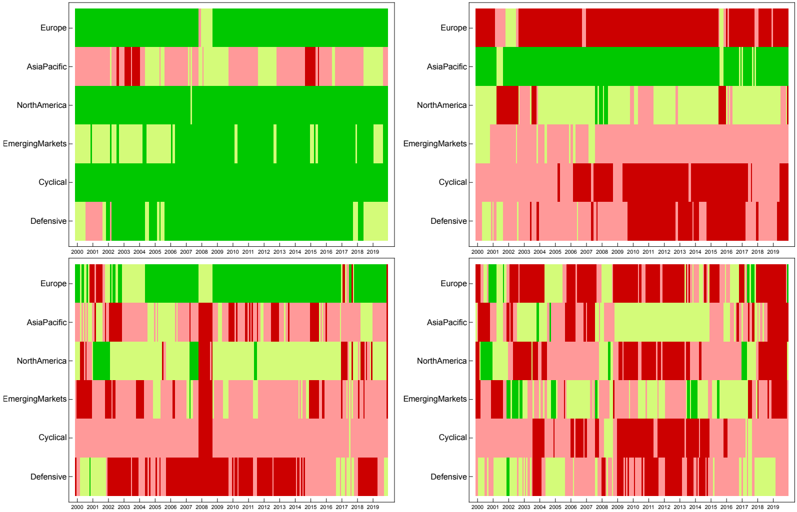
PC interpretation







PC interpretation



Some observations

- Procedure selects into appropriate basket, but does not indicate strength of correlation (e.g. May 2012 vs. Oct 2017).
- For strength, consider eigenvalue.
- First PC is always a global risk factor, often ex-Asia-Pacific.
- Cyclical industries are always strong in global factor; defensive industries less strong.
- Second PC is Asia-Pacific factor, mostly with North America.
- Third PC is typically Europe with North America.

Stress testing with aggregated risk factors

- Global stress scenario: adjust first PC or first two PCs, e.g. by choosing an explicit historical scenario or a historical realisation at a specific quantile.
- Asia-Pacific scenario: adjust second PC
- European scenario: adjust first and third PC
- North America scenario: adjust first and second PC
- Scenario "global economy more (less) connected": choose historical scenario where first PC's loadings are high (low)

Overview

Conclusion

- Factor models are used in various finance applications e.g. to estimate high-dimensional covariance matrices or in stress testing.
- Principal component analysis on a multivariate data set yields a factor model with latent factors.
- This is considered an unsupervised learning method.
- We attempt to give PCs on a data set consisting of risk factors (geographic regions and industries) an interpretation.
- Possible applications:
  - increase range of stress test scenarios
  - further decrease number of factors required for robust covariance matrix estimation

Outlook

- Possibly of interest: Alternative methods find relevant factors across a number of PCs (e.g. (Mao, 2005; Masaeli *et al.*, 2010; Enki *et al.*, 2013; Chang *et al.*, 2016).
- Possibly use Varimax instead of PCA (Kaiser, 1958). Varimax attempts to find axes with few large loadings and many near-zero loadings.
- Non-linear relationships: Kernel-PCA, Autoencoder.

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Thank you!

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