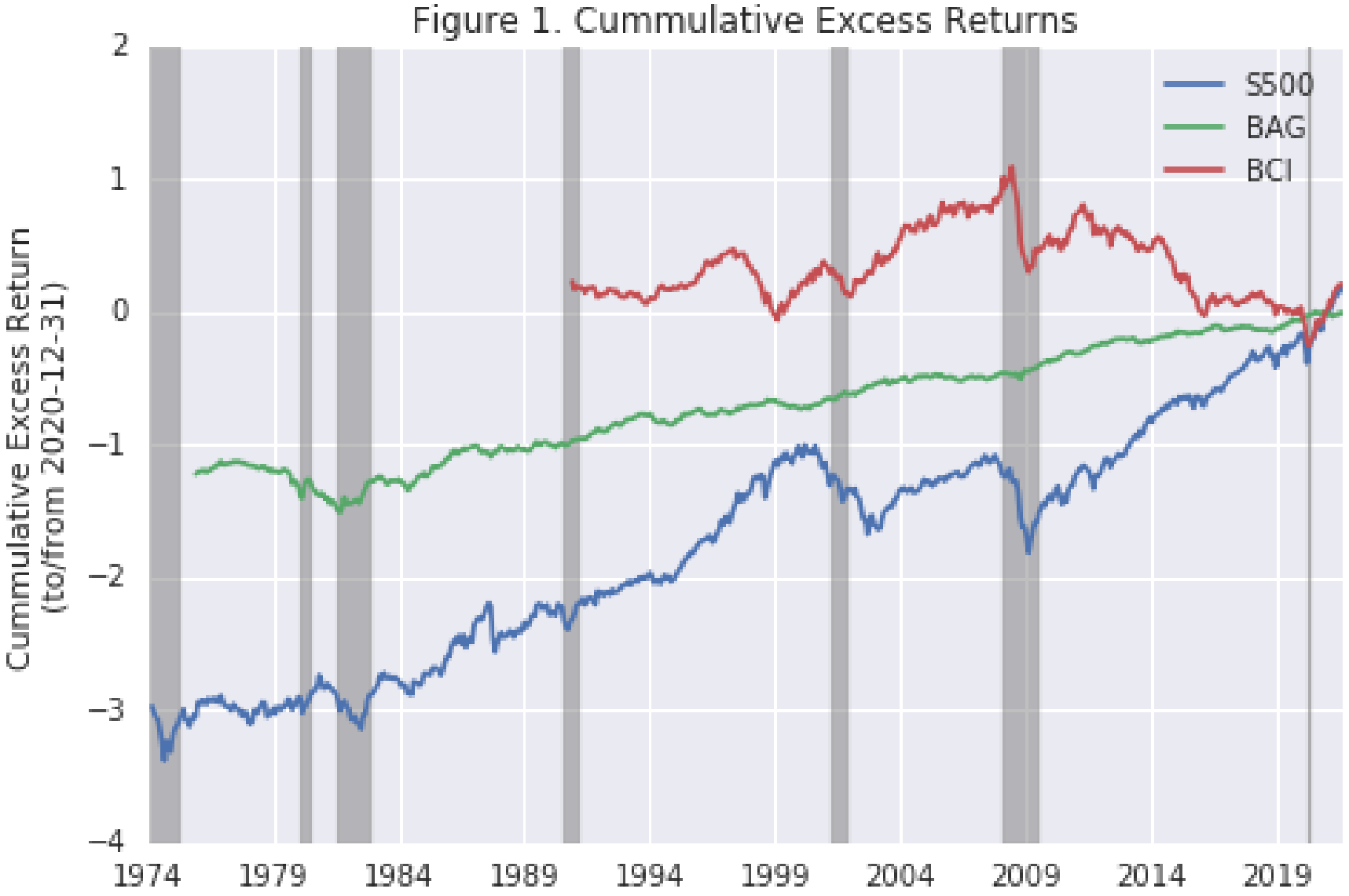
PIMCO

Fat Tailed Kelly

Steve Schulist

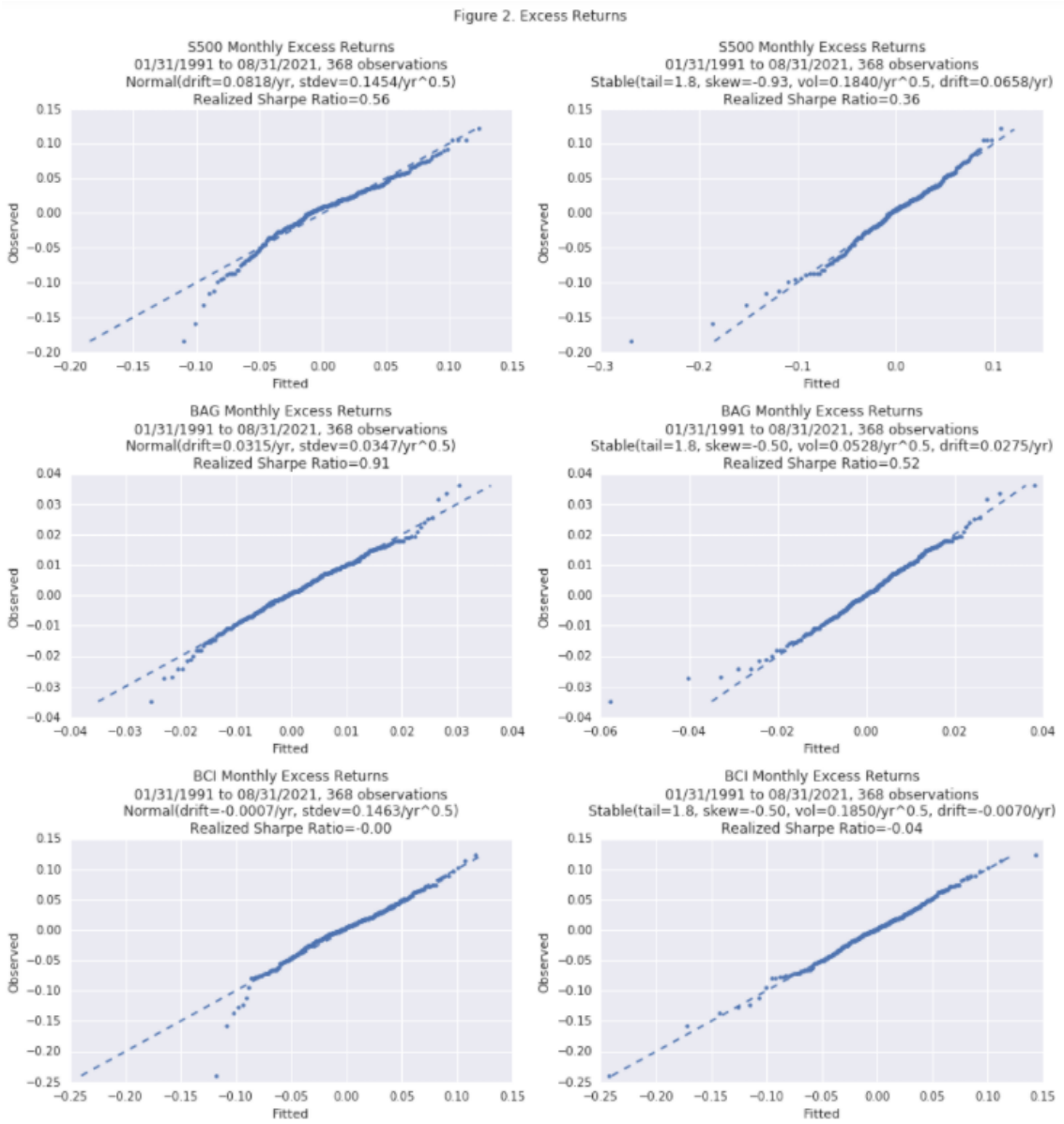
A company of Allianz

What asset allocation maximizes portfolio expected return?



Sources: S&P, Bloomberg

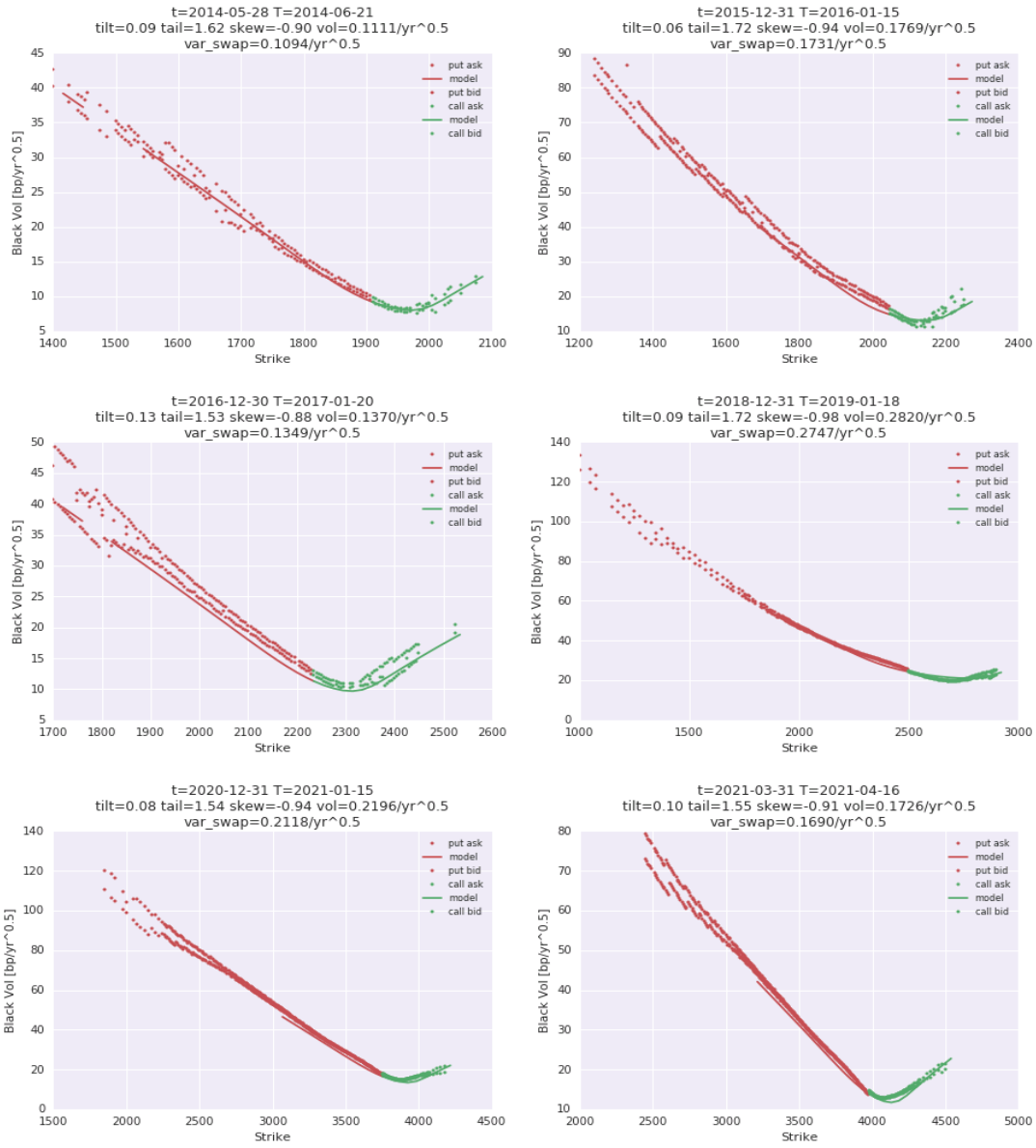
Backward looking history tells us returns have fat tails.



Sources: S&P, Bloomberg, the author

Forward looking markets tell us returns have fat tails.

Figure 3. Implied Vols



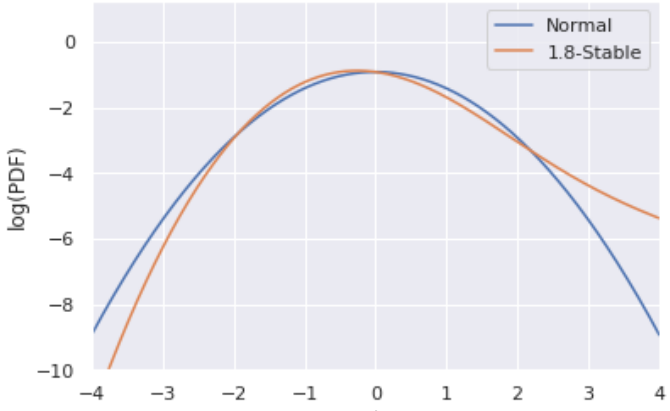
Sources: Bloomberg, the author

Math Outline

1. $T(R|P)$
2. $\mathcal{E}(\tilde{R}) = \exp(R)$
3. $\tilde{R}_t^{\xi} = \xi^T \cdot \tilde{R}_t + (1 - \xi^T \cdot \underline{1}) r_0 t$
4. No arbitrage
5. $\lambda(\xi) = \tilde{F}^{\xi}((-\infty, -1])$
6. $\exp(R^{\xi}) = \mathcal{E}(\tilde{R}^{\xi})$
7. $E_P\left(R_t^{\xi} \middle| \lambda = 0\right)$
8. ξ^*

Source: the author

Normal vs Stable



Normal

$$T(R|P) = (rt, \Sigma t, 0)$$

$$c \epsilon \mathbb{R}^M \times \mathbb{R}^N$$

$$\Sigma = c^T \cdot c$$

$$c^T \cdot \theta = \mu - r_0$$

$$\lambda = 0$$

$$c \cdot \xi^* = \theta$$

Stable

$$T(R|P) = (bt, 0, Ft)$$

$$c \epsilon \mathbb{R}^{2M} \times \mathbb{R}^N$$

$$d\Lambda(u) = \sum_{m=1}^{2M} |c_m|^\alpha \delta(u - u_m) du$$
$$c_m = c^T \cdot \hat{e}_m$$
$$u_m = c_m / |c_m|$$

$$c^T \cdot \theta = r - r_0$$

$$\lambda \geq 0$$

$$b - r_0 \underline{1} + \mathrm{E}_F(f) = \underline{0}$$
$$f(x) = (e^x - 1)/(1 + \xi^{*T} \cdot (e^x - 1)) - h(x) \in \mathbb{R}^N$$
$$\mathrm{E}_F(f) = a \sum_{m=1}^{M} |c_m|^\alpha \int_0^\infty \frac{d\rho}{\rho^{\alpha+1}} f(\rho u_m)$$

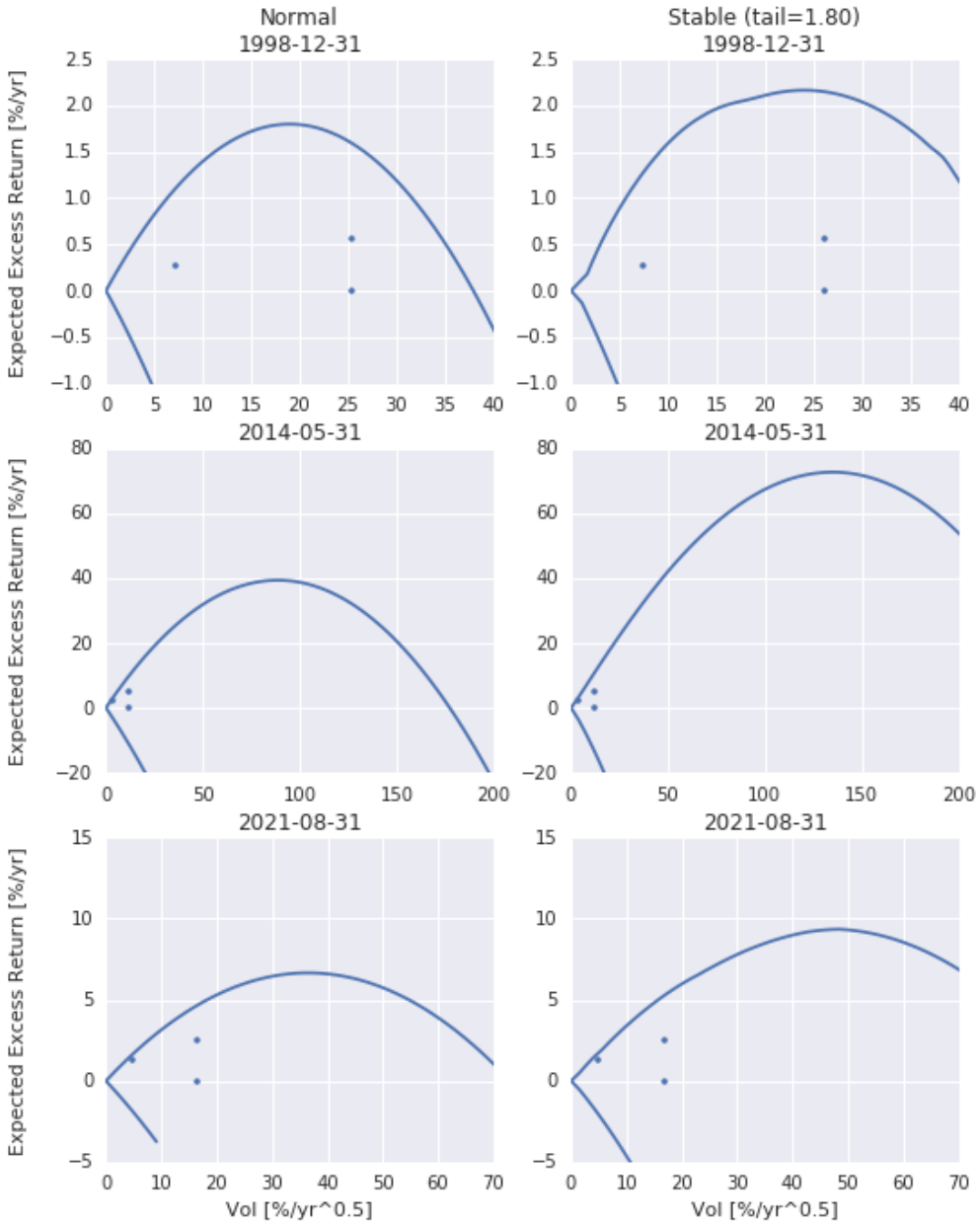
Source: the author

Market Environment

Figure 4. Market Environment

Source: Bloomberg

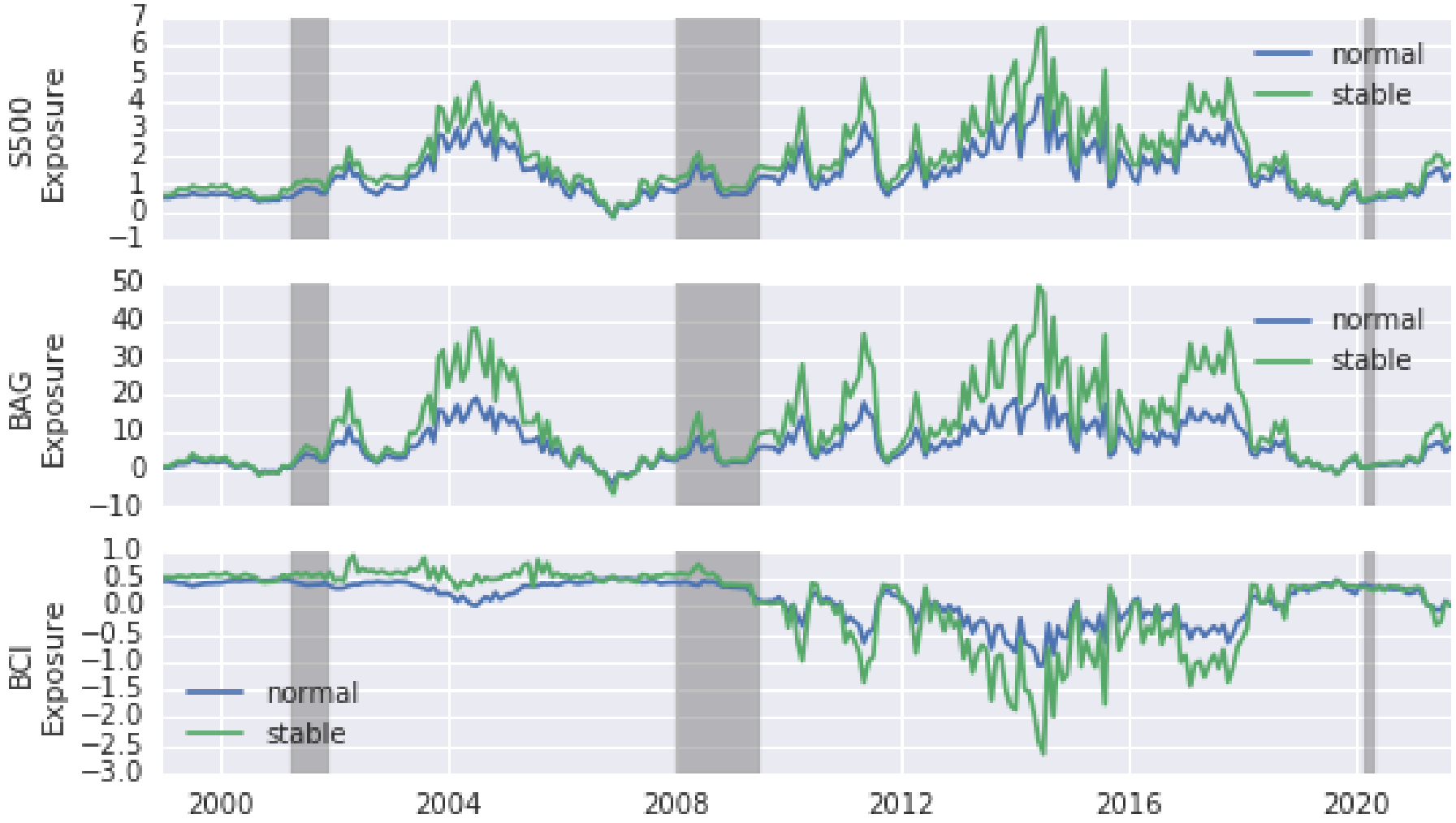
Efficient Frontiers



Source: the author
Hypothetical example for illustrative purposes only

Asset Allocation

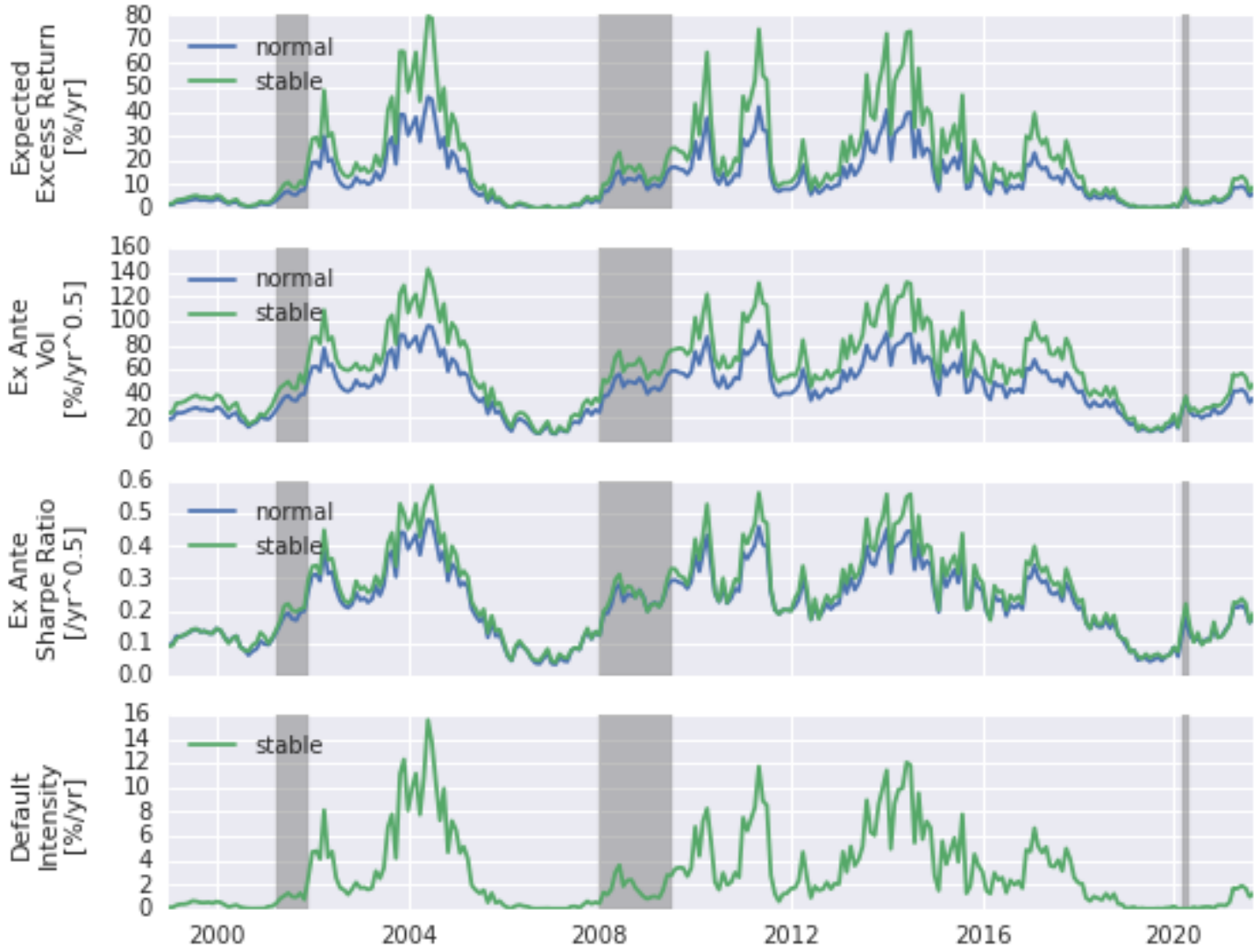
Figure 5. Kelly Implied Asset Exposures



Source: the author

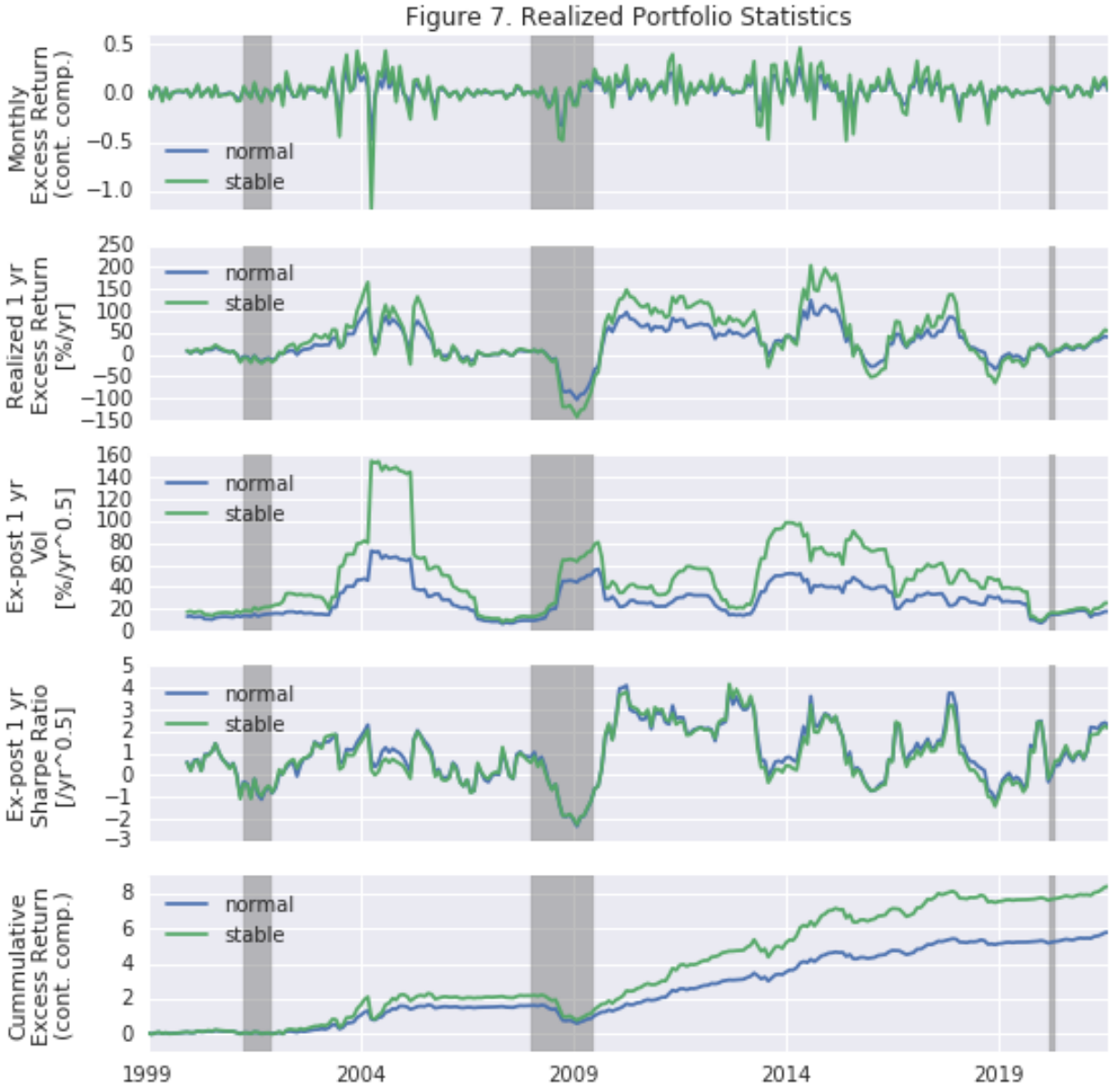
Risk Measures

Figure 6. Forward Looking Portfolio Statistics



Source: the author

Performance



Source: the author

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Appendix

This presentation is based on the article Fat Tail Kelly by Steve Schulist, published by Wilmott Magazine.

The paper and this presentation contains hypothetical analysis. Results shown may not be attained and should not be construed as the only possibilities that exist. The analysis reflected in this information is based on a set of assumptions believed to be reasonable at the time of creation. Actual returns will vary. Forecasts, estimates, and certain information contained herein are based on proprietary research and should not be considered as investment advice or a recommendation of any particular security, strategy, or investment product.

HYPOTHETICAL PERFORMANCE RESULTS HAVE MANY INHERENT LIMITATIONS, SOME OF WHICH ARE DESCRIBED BELOW. NO REPRESENTATION IS BEING MADE THAT ANY ACCOUNT WILL OR IS LIKELY TO ACHIEVE PROFITS OR LOSSES SIMILAR TO THOSE SHOWN. IN FACT, THERE ARE FREQUENTLY SHARP DIFFERENCES BETWEEN HYPOTHETICAL PERFORMANCE RESULTS AND THE ACTUAL RESULTS SUBSEQUENTLY ACHIEVED BY ANY PARTICULAR TRADING PROGRAM. ONE OF THE LIMITATIONS OF HYPOTHETICAL PERFORMANCE RESULTS IS THAT THEY ARE GENERALLY PREPARED WITH THE BENEFIT OF HINDSIGHT. IN ADDITION, HYPOTHETICAL TRADING DOES NOT INVOLVE FINANCIAL RISK, AND NO HYPOTHETICAL TRADING RECORD CAN COMPLETELY ACCOUNT FOR THE IMPACT OF FINANCIAL RISK IN ACTUAL TRADING. FOR EXAMPLE, THE ABILITY TO WITHSTAND LOSSES OR TO ADHERE TO A PARTICULAR TRADING PROGRAM IN SPITE OF TRADING LOSSES ARE MATERIAL POINTS WHICH CAN ALSO ADVERSELY AFFECT ACTUAL TRADING RESULTS. THERE ARE NUMEROUS OTHER FACTORS RELATED TO THE MARKETS IN GENERAL OR TO THE IMPLEMENTATION OF ANY SPECIFIC TRADING PROGRAM WHICH CANNOT BE FULLY ACCOUNTED FOR IN THE PREPARATION OF HYPOTHETICAL PERFORMANCE RESULTS AND ALL OF WHICH CAN ADVERSELY AFFECT ACTUAL TRADING RESULTS.

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