Risk management of insurance companies, pension funds and hedge funds using stochastic programming asset-liability models

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Outline

• This talk is a tutorial on asset liability models for pension funds, insurance companies,
  Bank trading departments and other institutions and individuals.
• Most situations are multiperiod, have risky assets to choose from and risky liability
  commitments to be satisfied along with various policy and instructional constraints.
• We will review various modeling approaches such as mean variance
  analysis, and continuous time modeling and show their advantages and limitations.
• But a better approach is through multiperiod stochastic programming modeling. This is scenario
  optimization which is a very useful practical approach that can handle the uncertain assets and liabilities
  and the policy and instructional constraints.
• The scenarios model how the future might evolve.
• We discuss various ways to estimate these scenarios and to determine which are most important for model
  success. There are economic, statistical, political and other considerations.
• We discuss the Russell Yasuda Kasai model for a large Japanese insurance company that won a major
  prize and was at its time the largest financial model. And we look at other models made by the Frank
  Russell Company designed by the author. These were successful models but were expensive to make.
• The InnoALM model for Siemens Austria was an improved model that was easier and less expensive to
  make. It was the only consistently used model for pensions and regulators in Austria.
• Convex risk measures are preferred, more easily understood, and safer than value at risk c-var and other
  measures.
• Scenario dependent correlation matrices are used and shown to be useful in all markets and preferable to
  average correlation matrices which will not work as they cannot capture bonds up stocks down in a stock
  market crash.
• We discuss other applications of the approach and compare it to other strategies such as mean variance,
  fixed mix, continuous time models and show their superiority.
• The discrete scenario stochastic programming approach is practical, relatively straightforward to implement
  and economically effective plus teaches the clients about their business in an easy to understand way,
Program

• I will discuss various approaches to asset liability management
• The one I prefer is scenario optimization through multiperiod stochastic programming
• Since I would like you to understand the approach and its advantages and disadvantages, I start with the ten things to remember
• This I discuss the other approaches, their advantages and disadvantages and conclude that a better approach is needed
• Then I cover the scenario optimization, stochastic programming approach via two very successful models and their possible applications.
• Finally we review the important things to remember.
The top ten points to remember about the stochastic programming approach to asset liability and wealth management.

**Point 1** - Means are by far the most important part of the distribution of returns – especially the direction.

→ You must estimate future means well or you can travel very fast in the wrong direction and that usually leads to losses or underperformance. The effect is risk aversion dependent. The less risk averse you are the bigger is the effect of mean estimation errors on performance.

The effect is risk aversion dependent. The less risk averse you are the bigger is the effect of mean estimation errors on performance.

**Point 2** - Mean variance models are useful as a basic guideline when you are in an assets only situation. Professionals adjust means using mean-reversion, James-Stein or truncated estimators and constrain output weights.

→ Do not change asset positions unless the advantage of the change is significant. Do not use mean variance analysis with liabilities and other major market imperfections except as a first test analysis.
Point 3 - Trouble arises when one overbets and a bad scenario occurs

→ You must not overbet when there is any possibility of a bad scenario occurring unless the bet is protected by some type of hedge or stop-loss. Mental stop-losses are often better than market ordered stops

Mental stop losses are often better than market ordered stops
Point 4 - Trouble is exacerbated when the expected diversification does not hold in the scenario that occurs

→ You must use scenario dependent correlation matrices because simulations around historical correlation matrices are inadequate for extreme scenarios.

Point 5 - When there is a large decline in the stock market, the positive correlation between stocks and bond fails and they become negatively correlated

→ When the mean of the stock market is negative, bonds are usually more attractive as is cash. So the correlation is negative.

Point 6 - Stochastic programming scenario based models are useful when one wants to look at aggregate overall decisions with liabilities, liquidity, taxes, policy, legal and other constraints and have targets and goals you want to achieve.

→ It pays to make a complex stochastic programming model when a lot is at stake and the essential problem has many complications.
Point 7 - Other approaches, continuous time finance, decision rule based SP, control theory, etc are useful for problem insights and theoretical results. But in actual use, they may lead to disaster unless modified. The Black-Scholes theory says you can hedge perfectly with log normal assets and this can lead to overbetting.

But fat tails and jumps arise frequently and can occur without warning. The S&P opened limit down –60 points or 6% when trading resumed after Sept 11, 2001 and it fell 14% that week.

→ Be careful of the assumptions, including implicit ones, of theoretical models. Use the results with caution no matter how complex and elegant the math or how smart or famous the author. Remember you have to be very smart to lose millions and even smarter to lose billions.
**Point 8** - Do not be concerned with getting all the scenarios exactly right when using stochastic programming models. You cannot do this and it does not matter that much anyway.

Rather worry that you have the problems’ periods laid out reasonably and the scenarios basically cover the means, the tails and the chance of what could happen. If the current situation has never occurred before, use one that is similar to add scenarios.

For a crisis in Brazil, use Russian crisis data, for example. The results of the SP will give you good advice when times are normal and keep you out of severe trouble when times are bad.

Those using SP models may lose 5-10-15% but they will not lose 50-70-95% like some investors and hedge funds.

→ If the scenarios are more or less accurate and the problem elements reasonably modeled, the SP will give good advice.

You may slightly outperform in normal markets but you will greatly outperform in bad markets when other approaches may blow up.

Current markets are also greatly affected by political concerns and statements by Central Bank officials and country lenders.
Point 9 - SP models for ALM were very expensive in the 1980s and early 1990s but are not very expensive now. Vancouver analysts using a large linear programming model to plan lumber operations at MacMillan Blodel used to fly to San Francisco to use a large computer which would run all day to run the model once. Now models of this complexity would take only seconds on an inexpensive desk top computer.

→ Advances in computing power and modeling expertise have made SP modeling not very expensive.

Such models are still complex and require approximately six months to develop and test, costing a couple hundred thousand dollars. A small team can now make a model for a complex organization quite quickly at fairly low cost compared to what is at stake.
Point 10 - Eventually as there are more disasters and more successful SP models are built and used, they will become popular.

→ The ultimate goal is to have them in regulations like VaR. While VaR does more good than harm, its safety is questionable in many applications. C-VaR is an improvement but for most people and organizations the non-attainment of goals is more than proportional, that is, convex in the non-attainment.

References:

• Ziemba, W.T. (2003), The Stochastic Programming Approach to Asset Liability and Wealth Management, AIMR, December, 192 pages plus 72-page appendix. [A review by Alan King is in Interfaces April 2006] Paperback on Amazon; free download from AIMR.


Books relevant to ALM management
Intertemporal Asset Management

- This is main street *academic finance*.
- Some use it, for example, DE Shaw Hedge Fund (so they told me during a talk in Zurich).
- It is not my choice for a practical, useful model of ALM. Allocations move like crazy and with log utility there is too much cash.
- But it is best to understand this approach, so we begin with it.

The Agenda

1. Basic setting
2. One period surplus management model
3. Intertemporal surplus management model
4. Risk preferences, funding ratio, and currency beta
5. Results

Reference:
Basic setting
Assumptions

• Pension Funds and life insurance companies have the legal order to invest in order to guarantee payments; the liabilities of such a company are determined by the present value of the payments.

• The growth of the value of the assets under management has to be orientated at the growth of the liabilities.

• Liabilities and assets are characterized by stochastic growth rates.

• Surplus Management: Investing assets such that the ratio between assets and liabilities always remains greater than one; i.e. such that the value of the assets exceeds the value of the liabilities in each moment of time.
Basic setting

References

- **Andrew D. Roy** (1952), *Econometrica*, the safety first principle, i.e. minimizing the probability for failing to reach a prespecified (deterministic) threshold return by utilizing the Tschbyscheff inequality


- **Robert C. Merton** (1993) in Clotfelter and Rothschild, "The Economics of Higher Education", optimization of a University's asset portfolios where the liabilities are given by the costs of its activities; the activity costs are modeled as state variables
Basic setting
References continuous time finance

• Robert C. Merton (1969 and 1973), The Review of Economics and Statistics and Econometrica, Introduction of the theory of stochastic processes and stochastic programming into finance, development of the intertemporal capital asset pricing model, the base for the valuation of contingent claims such as derivatives

One period surplus management model

Notation

The variance of the asset portfolio
\[ \sigma_A^2 \]

The variance of the liabilities
\[ \sigma_L^2 \]

The covariance between the asset portfolio and the liabilities:
\[ \sigma_{AL} \]

The vector of portfolio fractions of the risky portfolio:
\[ \omega' = (\omega_1, \ldots, \omega_n) \]

The vector of expected asset returns:
\[ \mu_A' = (E_1, \ldots, E_n) \]

The covariance matrix of the assets:
\[ V = \begin{pmatrix} \sigma_{11} & \cdots & \sigma_{1n} \\ \vdots & \ddots & \vdots \\ \sigma_{n1} & \cdots & \sigma_{nn} \end{pmatrix} \]

The vector of covariances between the assets and the liabilities:
\[ V_{AL} = (\sigma_{1L}, \ldots, \sigma_{nL}) \]

The unity vector:
\[ e' = (1, \ldots, 1) \in \mathbb{R}^n \]
One period surplus management model
Basic setting

Goal of the model: Identify an asset portfolio (consisting out of \( n \) risky and one riskless asset) which reveals a minimum variance of the surplus return

The \textit{surplus return} definition according to Sharpe and Tint (1990):

\[
\bar{R}_{St} = \frac{\tilde{S}_{t+1} - S_t}{A_t} = \left( \frac{\tilde{A}_{t+1} - \tilde{L}_{t+1}}{A_t} \right) \left( A_t - L_t \right) = \left( \frac{\tilde{A}_{t+1} - A_t}{A_t} \right) L_t \left( \frac{\tilde{L}_{t+1} - L_t}{A_t L_t} \right) = \bar{R}_{At} - \frac{1}{F_t} \bar{R}_{Lt}
\]

The \textit{expected surplus return}:

\[
E(\bar{R}_S) = E_A - \frac{1}{F} E_L
\]

The \textit{surplus variance}:

\[
\text{Var}(\bar{R}_S) = \sigma_A^2 + \frac{1}{F^2} \sigma_L^2 - 2 \cdot \frac{1}{F} \sigma_{AL}
\]
One period surplus management model
Basic setting

The expected surplus return in terms of assets returns:

\[ E(\tilde{R}_S) = E_S = \omega'(\mu_A - re) + r - \frac{1}{F} E_L \]

The variance of the surplus return in terms of asset returns:

\[ \text{Var}(\tilde{R}_S) = \sigma^2_S = \omega'V\omega + \frac{1}{F^2} \sigma^2_L - 2 \frac{1}{F} \cdot \omega'V_{AL} \]

The Lagrangian:

\[ L = \omega'V\omega + \frac{1}{F^2} \sigma^2_L - 2 \frac{1}{F} \cdot \omega'V_{AL} - \lambda \left( \omega'(\mu_A - re) + r - \frac{1}{F} E_L - E_S \right) \]

The optimum condition in a one period setting:

\[ \omega = \frac{1}{F} V^{-1}V_{AL} + \frac{\lambda}{2} V^{-1}(\mu_A - re) \]
**One period surplus management model**

Basic setting

Interpretation of the result:

The optimum portfolio consists out of two portfolios:

1. \( V^{-1}(\mu_A - re) \) This is the tangency portfolio in the CAPM framework

2. \( V^{-1}V_{AL} \) This is minimum surplus return variance portfolio

The concentration on one of these portfolios is dependent on the Lagrange multiplier \( \lambda \). This is the grade of appreciation of an additional percent of expected surplus return in terms of additional surplus variance (a risk aversion factor: how much additional risk is an investor willing to take for an additional percent of return).

Usual portfolio theory results in the context of surplus management.
**Intertemporal surplus management model**

**Objective function**

Goal of the model: Maximize the lifetime expected utility of a surplus management policy. It is assumed that the input parameters of the model fluctuate randomly in time depending on a state variable $Y$.

The **asset and the liability returns** follow Itô-processes: standard Wiener processes

\[
\tilde{R}_A = \frac{d\tilde{A}}{A} \equiv E_A(Y, t) dt + \sigma_A(Y, t) \cdot d\tilde{z}_A \\
\tilde{R}_L = \frac{d\tilde{L}}{L} \equiv E_L(Y, t) dt + \sigma_L(Y, t) \cdot d\tilde{z}_L
\]

\[
d\tilde{z}_A = \tilde{z}_A \sqrt{dt} \\
d\tilde{z}_L = \tilde{z}_L \sqrt{dt}
\]

The **state variable** follows a geometric Brownian motion:

\[
\frac{d\tilde{Y}}{Y} \equiv E_Y dt + \sigma_Y d\tilde{z}_Y \\
\tilde{z}_Y \equiv \tilde{z}_Y \sqrt{dt}
\]

This setting is equivalent to Merton (1973).
Intertemporal surplus management model
Transformations

Substituting these definitions into the definition for the surplus return yields:

\[
\tilde{R}_S = \frac{d\tilde{S}}{A} = \frac{d\tilde{A}}{A} - \frac{1}{F} \frac{d\tilde{L}}{L}
\]

\[
= \left( E_A(Y,t) - \frac{1}{F} E_L(Y,t) \right) dt + \left( \sigma_A(Y,t) \tilde{z}_A - \frac{1}{F} \sigma_L(Y,t) \tilde{z}_L \right) \sqrt{dt}
\]

\[
E(d\tilde{S}) = \left( E_A - \frac{1}{F} E_L \right) \cdot A \cdot dt
\]

\[
E(d\tilde{S}^2) = E \left[ \left( E_A - \frac{1}{F} E_L \right) \cdot A \cdot dt + \left( \sigma_A \tilde{z}_A - \frac{1}{F} \sigma_L \tilde{z}_L \right) A \sqrt{dt} \right]^2
\]

\[
= E \left[ \left( \sigma_A \tilde{z}_A - \frac{1}{F} \sigma_L \tilde{z}_L \right)^2 \right] \cdot A^2 \cdot dt \quad \text{weil } dt^2 = dt^{3/2} = 0
\]

\[
= \left( \sigma_A^2 \cdot E\left(\tilde{z}_A^2\right) + \frac{1}{F^2} \sigma_L^2 \cdot E\left(\tilde{z}_L^2\right) - \frac{1}{F} \cdot 2 \cdot \sigma_A \cdot \sigma_L \cdot E(\tilde{z}_A \cdot \tilde{z}_L) \right) \cdot A^2 \cdot dt
\]
**Intertemporal surplus management model**

Transformations

Because $E(\tilde{Z}_A) = E(\tilde{Z}_L) = 0$ and $E(\tilde{Z}_A^2) = E(\tilde{Z}_L^2) = 1$ and

$\sigma_A \cdot \sigma_L \cdot E(\tilde{Z}_A \cdot \tilde{Z}_L) = E(\tilde{R}_A \cdot \tilde{R}_L) = E[(\tilde{R}_A - E_A)(\tilde{R}_L - E_L)] = \sigma_{AL}$ follows:

$$E(d\tilde{S}^2) = \left(\sigma_A^2 + \frac{1}{F^2} \sigma_L^2 - 2 \cdot \frac{1}{F} \sigma_{AL}\right) \cdot A^2 \cdot dt$$

Furthermore, analogously:

$$E(d\tilde{S} \cdot d\tilde{Y}) = AY\sigma_A\sigma_Y E(\tilde{Z}_A \cdot \tilde{Z}_Y) dt - \frac{A}{F} Y\sigma_L\sigma_Y E(\tilde{Z}_L \cdot \tilde{Z}_Y) dt$$

$$= \left(\sigma_{AY} - \frac{1}{F} \sigma_{LY}\right) \cdot A \cdot Y \cdot dt$$

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**Intertemporal surplus management model**

**J - function**

The **J-function** is the expected lifetime utility in the decision period $t$ ...

\[
J(S,Y,t) \equiv \max_{\omega} E_t \left( \int_t^T U(S,Y,\tau) d\tau \right) = \max_{\omega} E_t \left( \int_t^{t+dt} U(S,Y,\tau) d\tau + \int_t^T U(S,Y,\tau) d\tau \right) \\
= \max_{\omega} E_t \left( \int_t^{t+dt} U(S,Y,\tau) d\tau + J \left( S + d\tilde{S}, Y + d\tilde{Y}, t + dt \right) \right) \\
= U(S,Y,t) dt + \max_{\omega} E_t \left[ J \left( S + d\tilde{S}, Y + d\tilde{Y}, t + dt \right) \right] \\
= U dt + \max_{\omega} E_t \left[ J + J_S d\tilde{S} + d\tilde{Y} J_Y + J_t dt + \frac{1}{2} J_{SS} d\tilde{S}^2 + \frac{1}{2} J_{YY} d\tilde{Y}^2 + J_{SY} d\tilde{S} d\tilde{Y} + o(dt) \right]
\]

by applying **Itô's lemma**. It follows the **fundamental partial differential equation** of intertemporal portfolio optimization:

\[
0 = U dt + \max_{\omega} E_t \left[ J_S d\tilde{S} + d\tilde{Y} J_Y + J_t dt + \frac{1}{2} J_{SS} d\tilde{S}^2 + \frac{1}{2} J_{YY} d\tilde{Y}^2 + J_{SY} d\tilde{S} d\tilde{Y} + o(dt) \right]
\]
**Intertemporal surplus management model**

Optimum condition

Substituting in the definitions of expected returns, variances and covariances:

\[ 0 = \max_{\omega} \left[ J_S \left( E_A - \frac{1}{F} E_L \right) \cdot A + J_Y E_Y + J_t + \frac{1}{2} J_{SS} \left( \sigma_A^2 + \frac{1}{F^2} \sigma_L^2 - 2 \frac{1}{F} \sigma_{AL} \right) \cdot A^2 \right. \]

\[ \left. + \frac{1}{2} J_{YY} \sigma_Y^2 \cdot Y^2 + J_{SY} \left( \sigma_{AY} - \frac{1}{F} \sigma_{LY} \right) \cdot A \cdot Y + U \right] \]

Substituting in the following definitions ...

\[ E_A = \omega' (u_A - re) + r \quad \sigma_A^2 = \omega' \sigma \omega \quad \sigma_{AL} = \omega' \sigma_{AL} \quad \sigma_{AY} = \omega' \sigma_{AY} \]

provides:

\[ 0 = \max_{\omega} \left[ J_S \left( \omega' (u_A - re) + r - \frac{1}{F} E_L \right) \cdot A + J_Y E_Y + J_t + \frac{1}{2} J_{SS} \left( \omega' \sigma \omega + \frac{1}{F^2} \sigma_L^2 - 2 \frac{1}{F} \omega' \sigma_{AL} \right) \cdot A^2 \right. \]

\[ \left. + \frac{1}{2} J_{YY} \sigma_Y^2 \cdot Y^2 + J_{SY} \left( \omega' \sigma_{AY} - \frac{1}{F} \sigma_{LY} \right) \cdot A \cdot Y + U \right] \]
**Intertemporal surplus management model**

Optimum portfolio weights

Differentiating this expression with respect to the vector of portfolio fractions $\omega$ yields ...

$$\omega = - \frac{J_S}{AJ_{SS}} V^{-1} (\mu_A - re) - \frac{Y J_{SY}}{AJ_{SS}} V^{-1} V_{AY} + \frac{1}{F} V^{-1} V_{AL}$$

$$= -a \frac{J_S}{AJ_{SS}} \omega_M - b \frac{Y J_{SY}}{AJ_{SS}} \omega_Y + c \frac{1}{F} \omega_L$$

where the following constants are defined:

$$a \equiv e' V^{-1} (\mu_A - re) \quad b \equiv e' V^{-1} V_{AY} \quad c \equiv e' V^{-1} V_{AL}$$
**Intertemporal surplus management model**

Four fund separation theorem

Maximizing lifetime expected utility of a surplus optimizer can be realized by investing in three portfolios:

1. The **market portfolio** which is the tangency portfolio in a classical CAPM framework:
   \[
   \omega_M \equiv \frac{V^{-1}(\mu_A - re)}{e'V^{-1}(\mu_A - re)}
   \]

2. The **hedge portfolio** for the state variable \(Y\) which corresponds to Merton’s (1973) intertemporal CAPM:
   \[
   \omega_Y \equiv \frac{V^{-1}V_{AY}}{e'V^{-1}V_{AY}}
   \]

3. A **hedge portfolio** for the fluctuations of the liabilities:
   \[
   \omega_L \equiv \frac{V^{-1}V_{AL}}{e'V^{-1}V_{AL}}
   \]
**Intertemporal surplus management model**

Interpretation

Classical result of Merton (1973): Both, the market portfolio and the hedge portfolio for the state variable, are hold in accordance to the risk aversion towards fluctuations in the surplus and the state variable.

New result of the intertemporal surplus management model: The weight of the hedge portfolio of the liability returns if $c/F$ which implies that all investors choose a hedging opportunity for the liabilities independent of their preferences. The only factor which influences this holding is the funding ratio of a pension fund.

The reason: All pension funds are influenced by wage fluctuations in the same way, whereas market fluctuations only affect such investors with high exposures in the market.
Intertemporal surplus management model

Why is $V^{-1}V_{AY}$ a hedge portfolio

Because this portfolio reveals the maximum correlation with the state variable:

$$\text{Cov} \left( \omega' \left( \begin{array}{c} \tilde{R}_1 \\ \vdots \\ \tilde{R}_n \\ \frac{d\tilde{Y}}{Y} \end{array} \right) \right) = \omega'V_{AY} \rightarrow \maxs.t. \omega'V\omega = \sigma_A^2$$

$$L = \omega'V_{AY} - \lambda \left( \omega'V\omega - \sigma_A^2 \right) \frac{\partial L}{\partial \omega} = V_{AY} - 2\lambda V\omega = 0 \iff \omega = \frac{1}{2\lambda} V^{-1}V_{AY}$$

which is identical with the hedge portfolio. Furthermore:

$$2\lambda V\omega = V_{AY} \iff 2\lambda \omega'V\omega = \omega'V_{AY} \iff 2\lambda = \frac{\sigma_{AY}}{\sigma_A^2} = \beta_{AY} \Rightarrow V^{-1}V_{AY} = \beta_{AY}\omega$$
Risk preferences, funding ratio, and currency beta

Four fund theorem

All investors hold four portfolios:
• The market portfolio
• The liabilities hedge portfolio
• One portfolio for each state variable
• The cash equivalent

The composition out of these portfolios depends on preferences, which are hardly interpretable.
Risk preferences, funding ratio, and currency beta
Different utility functions

Assumption of HARA-utility function (\(U \subset HARA \iff J \subset HARA\))

\[ U(S,Y,t) = \frac{S^\alpha}{\alpha} \Rightarrow J(S,Y,t) = \max_{\omega} E_t \left( \int_t^T \frac{S(\tau)^\alpha}{\alpha} d\tau \right), \]  

where \( S=S(\tau) = S[A(Y,\tau)] \)

Note that this implies the class of log-utility for \( \alpha \) approaching 0:

\[
\lim_{\alpha \to 0} U(S,Y,t) = \lim_{\alpha \to 0} \frac{S^\alpha}{\alpha} = \lim_{\alpha \to 0} \left( S^\alpha \cdot \ln S \right) \ln S
\]

Under this assumption we have:

\[
J_S = S^{\alpha-1}J_{SS} = (\alpha - 1) \cdot S^{\alpha-2}
\]

\[
J_{SY} = \frac{dS[A(Y)]^{\alpha-1}}{dY} = J_{SS} \cdot \frac{dS}{dA} \cdot \frac{dA}{dY} = J_{SS} \cdot \frac{dA}{dY}.
\]
Risk preferences, funding ratio, and currency beta
Portfolio holdings

The holdings of the market portfolio are:

$$-\frac{J_S}{A \cdot J_{SS}} = -\frac{S}{A(\alpha - 1)} = \frac{1}{1 - \alpha} \cdot \left(1 - \frac{1}{F}\right)$$

which simplifies in the log utility case to:

$$-\frac{J_S}{A \cdot J_{SS}} = -\frac{S}{A(\alpha - 1)} = 1 - \frac{1}{F}$$

The holdings of the state variable hedge portfolio are:

$$-\frac{Y \cdot J_{SY}}{A \cdot J_{SS}} = -\frac{dA}{A} \cdot \frac{dY}{Y} = -\frac{\tilde{R}_A}{\tilde{R}_Y}$$

The holdings of the liability hedge portfolio is \textit{independent of the class of utility function}. 
Risk preferences, funding ratio, and currency beta

Portfolio holdings

The portfolio holdings:

1. The *market portfolio* by the amount of
   \[-a \cdot \frac{J_S}{A \cdot J_{SS}} = a \cdot \frac{S}{A} = a \cdot \left(1 - \frac{1}{F}\right)\]

2. The *liability hedge* portfolio by the amount of
   \[\frac{c}{F}\]

3. The *hedge portfolio* by the amount of:
   \[-b \cdot \frac{Y \cdot J_{SY}}{A \cdot J_{SS}} = -b \cdot Y \cdot \left[\frac{1}{S(A(Y))} \frac{dA}{dY}\right] = -b \cdot \frac{dA}{dY} = -b \cdot \frac{dA}{dY} = -b \cdot \frac{\tilde{R}_A}{\tilde{R}_Y}\]

4. The *cash equivalent portfolio*
Risk preferences, funding ratio, and currency beta

Hedge ratio and portfolio holdings

Assuming the regression model without constant provides:

\[ \tilde{R}_A = \beta (\tilde{R}_A, \tilde{R}_Y) \tilde{R}_Y \Rightarrow \beta (\tilde{R}_A, \tilde{R}_Y) = \frac{\tilde{R}_A}{\tilde{R}_Y} \]

\[ - \beta (\tilde{R}_A, \tilde{R}_Y) = - \frac{E(\tilde{R}_A \cdot \tilde{R}_Y)}{E(\tilde{R}_Y^2)} = - \frac{\sigma_{AY}}{\sigma_Y^2} \]

\[ - b \cdot \frac{Y \cdot J_{SY}}{A \cdot J_{SS}} = - b \cdot \frac{\tilde{R}_A}{\tilde{R}_Y} = - b \cdot \beta (R_A, R_Y) = - b \cdot \frac{\sigma_{AY}}{\sigma_Y^2} \]

The risk tolerance against the state variable equals the negative hedge ratio of the portfolio against the state variable. If the state variable is an exchange rate, the risk tolerance equals the negative currency hedge ratio.
**Risk preferences, funding ratio, and currency beta**

Hedge ratio and portfolio holdings

**Assumption 1**: The state variable is the exchange rate risk

**Assumption 2**: There are $k$ foreign currency exposures. Each foreign currency is represented by a state variable.
Risk preferences, funding ratio, and currency beta
Hedge ratio and portfolio holdings

The following modifications have to be implemented:

(a) The state variables with returns:
\[ \tilde{R}_Y, \ldots, \tilde{R}_{Y_k} \]

(b) The risk tolerances for state variables:
\[ \frac{Y_1 J_{SY_1}}{AJ_{SS}} = -\beta(\tilde{R}_A, \tilde{R}_{Y_1}), \ldots, \frac{Y_k J_{SY_k}}{AJ_{SS}} = -\beta(\tilde{R}_A, \tilde{R}_{Y_k}) \]

(c) The covariances between the state variables and the assets:
\[ V_{AY_1}, \ldots, V_{AY_k} \]

(d) The hedge portfolios:
\[ \omega_{Y_1} = \frac{V^{-1}V_{AY_1}}{e'V^{-1}V_{AY_1}}, \ldots, \omega_{Y_k} = \frac{V^{-1}V_{AY_k}}{e'V^{-1}V_{AY_k}} \]

(e) The b-coefficients:
\[ b_1 = e'V^{-1}V_{AY_1}, \ldots, b_k = e'V^{-1}V_{AY_k} \]
Substituting these expressions into the equation for the portfolio allocation provides the **optimum asset holdings of an internationally diversified pension fund** ...

\[
\omega = a\left(1 - \frac{1}{F}\right)\omega_M - \sum_{i=1}^{k} b_i \beta(R_A, R_{Y_i})\omega_{Y_i} + c \frac{1}{F}\omega_L
\]

where

\[
\beta(R_A, R_{Y_i}) = \sum_{I=1}^{n} \omega_{I} \beta(R_{A_I}, R_{Y_i})
\]

**Problem:** The portfolio beta depends on the asset allocation vector \(\omega\). No analytical solution is possible.

**Solution:** Approximating the portfolio weights by a numerical procedure.
### Case study

#### Descriptive statistics

<table>
<thead>
<tr>
<th></th>
<th>Mean return</th>
<th>Volatility</th>
<th>Beta GBP</th>
<th>Beta JPY</th>
<th>Beta EUR</th>
<th>Beta CAD</th>
<th>Beta CHF</th>
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<tbody>
<tr>
<td><strong>Stocks</strong></td>
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<td>0.13</td>
<td>-0.32</td>
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<td>0.00</td>
<td>-0.06</td>
<td>-0.04</td>
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<td>-0.99</td>
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<tr>
<td><strong>Exchange rates in USD</strong></td>
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<td></td>
<td></td>
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<td></td>
</tr>
<tr>
<td>GBP</td>
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<td>JPY</td>
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<td>EUR</td>
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<td>0.1</td>
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</tr>
<tr>
<td>CAD</td>
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<td>0</td>
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<tr>
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<td>0.52</td>
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<tr>
<td>Wages and salaries</td>
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<td>0.01</td>
<td>0</td>
<td>-0.01</td>
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</table>

The stock market data are based on MSCI indices and the bond data on JP Morgan indices (Switzerland on Salomon Brothers data). The wage and salary growth rate is from Datastream. Monthly data between January 1987 and July 2000 (163 observations) is used. All coefficients are in USD. The average returns and volatilities are in percent per annum.

*ECU before January 1999.
Case study

Optimum portfolios of an internationally diversified pension fund from a USD perspective

<table>
<thead>
<tr>
<th></th>
<th>Market portfolio</th>
<th>Liability hedge portfolio</th>
<th>Hedge portfolio GBP</th>
<th>Hedge portfolio JPY</th>
<th>Hedge portfolio EUR</th>
<th>Hedge portfolio CAD</th>
<th>Hedge portfolio CHF</th>
</tr>
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<td>−31.0</td>
<td>−1.1</td>
<td>−8.6</td>
<td>−85.3</td>
<td>1.9</td>
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<td>−68.3</td>
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<td>23.5</td>
<td>138.3</td>
<td>−3.8</td>
</tr>
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<td>5.1</td>
<td>14.6</td>
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<td>−14.6</td>
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<td></td>
<td></td>
</tr>
<tr>
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<td>126.0</td>
<td>−126.4</td>
<td>−28.6</td>
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<td>133.9</td>
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<td>−30.1</td>
<td>6.4</td>
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<tr>
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<td>−32.6</td>
<td>−7.9</td>
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<td>42.4</td>
<td>62.9</td>
<td>143.8</td>
<td>110.7</td>
</tr>
</tbody>
</table>

The portfolio holdings are based on Eq. (9). All portfolio fractions are percentages. A riskless rate of interest of 2% per annum is assumed.
**Case study: too much cash!**

<table>
<thead>
<tr>
<th>Weightings of the funds due to different funding ratios</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Funding ratio</strong></td>
</tr>
<tr>
<td>Market portfolio (%)</td>
</tr>
<tr>
<td>Liability hedge portfolio (%)</td>
</tr>
<tr>
<td>Hedge portfolio GBP (%)</td>
</tr>
<tr>
<td>Hedge portfolio JPY (%)</td>
</tr>
<tr>
<td>Hedge portfolio EUR (%)</td>
</tr>
<tr>
<td>Hedge portfolio CAD (%)</td>
</tr>
<tr>
<td>Hedge portfolio CHF (%)</td>
</tr>
<tr>
<td>Riskless assets (%)</td>
</tr>
</tbody>
</table>

| Portfolio beta against GBP | -0.01 | -0.01 | -0.01 | -0.01 | -0.01 | -0.01 |
| Portfolio beta against JPY | 0.02 | 0.02 | 0.02 | 0.02 | 0.02 | 0.02 |
| Portfolio beta against EUR | 0.01 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| Portfolio beta against CAD | -0.02 | -0.02 | -0.01 | -0.01 | -0.01 | -0.01 |
| Portfolio beta against CHF | 0.02 | 0.01 | 0.01 | 0.01 | 0.01 | 0.01 |

The weightings of the portfolios according to Eq. (16), where $\alpha = 0$, i.e. log utility, is assumed.
Appendix: Basic relationships

\[ E(dS) = A(t) \left( \mu_A - \frac{\mu_L}{F(t)} \right) \cdot dt \]

\[ dS(t) \cdot dS(t) = A^2(t) \left( \sigma_A^2 + \frac{\sigma_L^2}{F^2(t)} - 2 \cdot \frac{P \cdot \sigma_A \cdot \sigma_L}{F(t)} \right) \cdot dt \]

\[ dS(t) \cdot dY(t) = A(t) \cdot Y(t) \left( \sigma_{Al} - \frac{P \cdot \sigma_L \cdot \sigma_Y}{F(t)} \right) \cdot dt \]

\[ dY \cdot dY = Y^2(t) \cdot \sigma_Y^2 \cdot dt, \]
Introduction to stochastic programming scenario optimization

- All individuals and institutions regularly face asset liability decision making.

- I discuss an approach using scenarios and optimization to model such decisions for pension funds, insurance companies, individuals, retirement, bank trading departments, hedge funds, etc.

- It includes the essential problem elements: uncertainties, constraints, risks, transactions costs, liquidity, and preferences over time, to provide good results in normal times and avoid or limit disaster when extreme scenarios occur.

- The stochastic programming approach while complex is a practical way to include key problem elements that other approaches are not able to model.

- Other approaches (static mean variance, fixed mix, stochastic control, capital growth, continuous time finance etc.) are useful for the micro analysis of decisions and the SP approach is useful for the aggregated macro (overall) analysis of relevant decisions and activities.

- It pays to make a complex stochastic programming model when a lot is at stake and the essential problem has many complications.
Other approaches - continuous time finance, capital growth theory, decision rule based SP, control theory, etc - are useful for problem insights and theoretical results.

They yield **good results most of the time** but frequently lead to the recipe for **disaster**: 

**over-betting and not being truly diversified at a time when an extreme scenario occurs.**

- BS theory says you can hedge **perfectly** with LN assets and this can lead to overbetting.
- But fat tails and jumps arise frequently and can occur without warning. The S&P opened limit down –60 or 6% when trading resumed after Sept 11 and it fell 14% that week.
- With derivative trading positions are changing constantly, and a non-overbet situation can become overbet very quickly.

Be careful of the **assumptions**, including implicit ones, of theoretical models. Use the results with **caution** no matter how complex and elegant the math or how **smart** the author.

Remember you have to be very smart to lose millions and even smarter to lose billions.
The uncertainty of the random return and other parameters is modeled using discrete probability scenarios that approximate the true probability distributions.

- The accuracy of the actual scenarios chosen and their probabilities contributes greatly to model success.
- However, the scenario approach generally leads to superior investment performance even if there are errors in the estimations of both the actual scenario outcomes and their probabilities.
- It is not possible to include all scenarios or even some that may actually occur. The modeling effort attempts to cover well the range of possible future evolution of the economic environment.
- The predominant view is that such models do not exist, are impossible to successfully implement or they are prohibitively expensive.
- I argue that give modern computer power, better large scale stochastic linear programming codes, and better modeling skills that such models can be widely used in many applications and are very cost effective.
Academic references:


• For an MBA level practical tour of the area


• If you want to learn how to make and solve stochastic programming models

• The case study at the end is based on Geyer and Ziemba (2008) The Innovest Austrian Pension Fund Planning Model InnoALM, *Operations Research*

• W T Ziemba (2016) An approach to financial planning of retirement pensions with scenario dependent correlation matrices and convex risk measures, *Journal of Retirement*
Mean variance models are useful as a basic guideline when you are in an assets only situation.

Professionals adjust means (mean-reversion, James-Stein, etc) and constrain output weights.

_Do not change asset positions unless the advantage of the change is significant._

_Do not use mean variance analysis with liabilities and other major market imperfections except as a first test analysis._
Mean Variance Models

Defines risk as a terminal wealth surprise regardless of direction
  • Makes no allowance for skewness preference
  • Treats assets with option features inappropriately

Two distributions with identical means and variances but different skewness
The Importance of getting the mean right. The mean dominates if the two distributions cross only once.

Thm: Hanoch and Levy (1969)

• If \(X \sim F(\cdot)\) and \(Y \sim G(\cdot)\) have CDF’s that cross only once, but are otherwise arbitrary, then \(F\) dominates \(G\) for all concave \(u\).

• The mean of \(F\) must be at least as large as the mean of \(G\) to have dominance.

• Variance and other moments are unimportant. Only the means count.

• With normal distributions \(X\) and \(Y\) will cross only once iff the variance of \(X\) does not exceed that of \(Y\).

• That’s the basic equivalence of Mean-Variance analysis and Expected Utility Analysis via second order (concave, non-decreasing) stochastic dominance.
Errors in Means, Variances and Covariances

Kallberg-Ziemba 1984, Chopra-Ziemba JPM 1993

Replace true mean \( \mu_i \) by \( \mu_i (1+kZ_i) \)

\[ Z_i \sim N(0,1) \]

size of error \( \rightarrow \) scale factor \( k = 0.05 \) to \( 0.20 \)

Replace true variances \( \sigma_i^2 \) by \( \sigma_i^2 (1+kZ_i) \)

Replace true covariances \( \sigma_{ij} \) by \( \sigma_{ij}^2 (1+kZ_i) \)

10 DJIA Securities 1980-89 Alcoa, Boeing, Coke, Dupont, Sears, etc

Monthly data

\[ u(CE) = E\xi u(\xi'x) \Rightarrow CE = u^{-1}[E\xi (\xi'x)] \]

Certainty equivalent \( CE = u^{-1} \) (expected utility of risky portfolio)

Measure CE Loss \( u = \) exponential, normal distributions \( \rightarrow \) mean-variance

exact formulas to compute \( \mu - \frac{1}{\tau} \sigma^2 \)

Certainty equivalent loss \( \Rightarrow CEL = \left( \frac{CE_{\text{act}} - CE_{\text{exp}}}{CE_{\text{exp}}} \right) \times 100 \)
Mean Percentage Cash Equivalent Loss Due to Errors in Inputs

Average Ratio of CEL for Errors in Means, Variances and Covariances

<table>
<thead>
<tr>
<th>t Risk Tolerance</th>
<th>Errors in Means vs Covariances</th>
<th>Errors in Means vs Variances</th>
<th>Errors in Variances vs Covariances</th>
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<tbody>
<tr>
<td>25</td>
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<td>3.22</td>
<td>1.67</td>
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<tr>
<td>50</td>
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<td>75</td>
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<tr>
<td>75</td>
<td>↓</td>
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<tr>
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<td>11.25</td>
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<tr>
<td></td>
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</table>

The error depends on the risk tolerance but roughly

<table>
<thead>
<tr>
<th>Error Mean</th>
<th>Error Var</th>
<th>Error Covar</th>
</tr>
</thead>
<tbody>
<tr>
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<td>2</td>
<td>1</td>
</tr>
</tbody>
</table>

Risk tolerance is the reciprocal of risk aversion.

When $R_A$ is very low such as with log $u$, then the errors in means become 100 times as important.

Conclusion: spend your money getting good mean estimates and use historical variances and covariances.
Average turnover: percentage of portfolio sold (or bought) relative to preceding allocation

- Moving to (or staying at) a near-optimal portfolio may be preferable to incurring the transaction costs of moving to the optimal portfolio.
- High-turnover strategies are justified only by dramatically different forecasts.
- There are a large number of near-optimal portfolios.
- Portfolios with similar risk and return characteristics can be very different in composition.

In practice (Frank Russell for example) only change portfolio weights when they change considerably 10, 20 or 30%.

• Optimization overweights (underweights) assets that are over(under) estimated
• Admits no tradeoff between short and long term goals
• Ignores the dynamism present in the world
• Cannot deal with liabilities
• Ignores taxes, transactions costs, etc
• Optimization treats means, covariances, variances as certain values when they are really uncertain in scenario analysis this is done better

So we reject variance as a risk measure for multiperiod stochastic programming models.

• But we use a distant relative – weighted downside risk from not achieving targets of particular types in various periods.
• We trade off mean return versus $R_A$ Risk so measured
Objective: maximize expected long run wealth at the horizon, risk adjusted. That is net of the risk cost of policy constraint shortfalls.

Problems are enormously complex.

Is it possible to implement such models that will really be successful?

*Impossible* said previous consultant [Nobel Laureate Bill Sharpe, now he’s more of a convert]

Models will sell themselves as more are built and used successfully.
Some possible approaches to model situations with such events

- Simulation: too much output to understand but very useful as check
- Mean Variance: ok for one period but with constraints, etc
- Expected Log: very risky strategies that do not diversify well. Fractional Kelly with downside constraints are excellent for risky investment betting
- Stochastic Programming/Stochastic Control: Mulvey does this (volatility pumping) with Decision Rules (eg Fixed Mix)
- Stochastic Programming: a very good approach

For a comparison of all these, see Introduction in ZM
Asset proportions: not practical
Stochastic Programming Approach - Ideally suited to Analyze Such Problems

- Multiple time periods; end effects - steady state after decision horizon adds one more decision period to the model

- Consistency with economic and financial theory for interest rates, bond prices etc

- Discrete scenarios for random elements - returns, liability costs, currency movements

- Utilize various forecasting models, handle fat tails

- Institutional, legal and policy constraints

- Model derivatives and illiquid assets

- Transactions costs
Stochastic Programming Approach - Ideally suited to Analyze Such Problems 2

- Expressions of risk in terms understandable to decision makers

- Maximize long run expected profits net of expected discounted penalty costs for shortfalls; pay more and more penalty for shortfalls as they increase (preferable to VaR)

- Model as constraints or penalty costs in objective maintain adequate reserves and cash levels meet regularity requirements

- Can now solve very realistic multiperiod problems on modern workstations and PCs using large scale linear programming and stochastic programming algorithms

Model makes you **diversify** – the key for keeping out of trouble
• 1950s fundamentals
• 1970s early models ≈ 1975 work with students Kusy and Kallberg
• early 1990s Russell-Yasuda model and its successors on work stations
• late 1990s ability to solve very large problems on PCs
• 2000+ mini explosion in application models

References


Stochastic Programming

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- Charnes & Cooper, 1959
- Chance-Constrained Programming
- Markowitz, 1952, 1959, 1987
- Mean Variance Portfolio Selection
- Merton, 1969, 1992
- Continuous Time Finance
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- Distribution Problems

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- Kallberg, White & Ziemba, 1982
- Kusy & Ziemba, 1986
- Dantzig, Infanger, 1991
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- Infanger, 1996
- Russell-Mitsubishi PALMS, 1995
- Mulvey, Torlacius & Wendt, 1995
- Dert, 1995
- King & Warden, Allstate, 1994, 1996
- Fannie Mae

Modern Models
- Zenios, 1991-1996
- Klassen, 1994
- Golub, Holmer, Zenios et al, 1994
- Mulvey & Vladimir, 1989, 1992
- Franendorfer and Schürle, 1996
- Cariño and Turner, 1996
- Hiller & Shapiro, 1989
- Shapiro, 1988
- Nielson & Zenios, 1992
- Merton, 1993
- Russell-Yasuda, 1994, 1995
- Berger & Mulvey 1996
- Boender, 1994
- Boender and Aalst, 1996
- Mulvey, Torlacius & Wendt, 1995
- Towers-Perrin, 1995
- Dantzig, Infanger, 1991
- Hiller & Eckstein, 1993
- MIDAS
- Hensel, Ezra and Ilkiw, 1991
- Wilkie, 1985-87
- Wilkie, 1995
- Brennan, Schwartz and Lagnado, 1993
- Wilkie, 1995
- Klassen, 1994
- Golub, Holmer, Zenios et al, 1994
- Mulvey & Vladimir, 1989, 1992
- Franendorfer and Schürle, 1996
- Cariño and Turner, 1996
- Hiller & Shapiro, 1989
- Shapiro, 1988
- Nielson & Zenios, 1992
- Merton, 1993
- Russell-Yasuda, 1994, 1995
- Berger & Mulvey 1996
- Boender, 1994
- Boender and Aalst, 1996
- Mulvey, Torlacius & Wendt, 1995
- Towers-Perrin, 1995
- Dantzig, Infanger, 1991
- Hiller & Eckstein, 1993
- MIDAS
- Hensel, Ezra and Ilkiw, 1991
- Wilkie, 1985-87
- Wilkie, 1995
- Brennan, Schwartz and Lagnado, 1993
- Wilkie, 1995
## ALM Models - Frank Russell

<table>
<thead>
<tr>
<th>Model</th>
<th>Type of Application</th>
<th>Year Delivered</th>
<th>Number of Scenarios Used</th>
<th>Computer Hardware Implementation</th>
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<td>Russell-Yasuda (Tokyo)</td>
<td>Property and Casualty Insurance</td>
<td>1991</td>
<td>256</td>
<td>IBM RISC 6000</td>
</tr>
<tr>
<td>Mitsubishi Trust (Tokyo)</td>
<td>Pension Consulting</td>
<td>1994</td>
<td>2000</td>
<td>IBM RISC 6000 with Parallel Processors</td>
</tr>
<tr>
<td>Swiss Bank Corporation (Basle)</td>
<td>Pension Consulting</td>
<td>1996</td>
<td>8000</td>
<td>IBM UNIX2</td>
</tr>
<tr>
<td>Daido Life Insurance Company (Tokyo)</td>
<td>Life Insurance</td>
<td>1997</td>
<td>25,600</td>
<td>IBM PC</td>
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<tr>
<td>Banca Fideuram (Rome)</td>
<td>Assets Only Personal</td>
<td>1997</td>
<td>10,000</td>
<td>IBM UNIX2 and PC</td>
</tr>
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</table>
Do not be concerned with getting all the scenarios exactly right when using stochastic programming models

You cannot do this and it does not matter much anyway.

Rather worry that you have the problems’ periods laid out reasonably and the scenarios basically cover the means, the tails and the chance of what could happen.

If the current situation has never occurred before, use one that’s similar to add scenarios. For a crisis in Brazil, use Russian crisis data for example. The results of the SP will give you good advice when times are normal and keep you out of severe trouble when times are bad.

Those using SP models may lose 5-10-15% but they will not lose 50-70-95% like some investors and hedge funds.

- If the scenarios are more or less accurate and the problem elements reasonably modeled, the SP will give good advice.
- You may slightly underperform in normal markets but you will greatly overperform in bad markets when other approaches may blow up.
Despite good results, fixed mix and buy and hold strategies do not utilize new information from return occurrences in their construction.

By making the strategy scenario dependent using a multi-period stochastic programming model, a better outcome is possible.

Example

- Consider a three period model with periods of one, two and two years. The investor starts at year 0 and ends at year 5 with the goal is to maximize expected final wealth net of risk.
- Risk is measured as one-sided downside based on non-achievement of a target wealth goal at year 5.
- The target is 4% return per year or 21.7% at year 5.
A shortfall cost function: target 4% a year

The penalty for not achieving the target is steeper and steeper as the non-achievement is larger.

For example, at 100% of the target or more there is no penalty, at 95-100% it’s a steeper, more expensive penalty and at 90-95% it’s steeper still.

This shape preserves the convexity of the risk penalty function and the piecewise linear function means that the stochastic programming model remains linear.

The concave objective function is

\[
\text{Maximize } E\left( \text{Final wealth} - \frac{\text{Accumulated penalized shortfalls}}{\text{Risk tolerance}} \right),
\]

where the risk tolerance equals the reciprocal of the Arrow–Pratt absolute risk-aversion index and balances risk and return.
Means, variances and covariances of six asset classes

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<td>U.S. large-cap equity</td>
<td>11.0%</td>
<td>17.0%</td>
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The scenarios are all the possible paths of returns that can occur over the three periods.

The goal is to make 4% each period so cash that returns 5.7% will always achieve this goal.

Bonds return 7.0% on average so usually return at least 4%.

But sometimes they have returns below 4%.

Equities return 11% and also beat the 4% hurdle most of the time but fail to achieve 4% some of the time.

Assuming that the returns are independent and identically distributed with lognormal distributions, we have the following twenty-four scenarios (by sampling 4x3x2), where the heavy line is the 4% threshold or 121.7 at year 5.
Scenarios

A. Cash

Wealth ($ millions)

Year

Wealth ($ millions)

Year

Wealth ($ millions)

Year
Scenarios in three periods
Example scenario outcomes listed by node

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<td>1</td>
<td>0.3333</td>
<td>0.032955</td>
<td>0.341701</td>
<td>0.041221</td>
<td>0.279216</td>
<td>0.027300</td>
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<td>2</td>
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<td>−0.091184</td>
<td>0.049939</td>
<td>0.109955</td>
<td>0.082171</td>
<td>−0.128904</td>
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<tr>
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<td>0.534592</td>
<td>0.120825</td>
<td>0.204917</td>
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<td>0.132663</td>
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<td>0.056592</td>
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<td>0.5000</td>
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<td>−0.130465</td>
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<tr>
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<td>0.054468</td>
<td>0.118764</td>
<td>−0.048986</td>
<td>0.065222</td>
<td>0.088793</td>
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We compare two strategies

1. the dynamic stochastic programming strategy which is the full optimization of the multiperiod model; and
2. the fixed mix in which the portfolios from the mean-variance frontier have allocations rebalanced back to that mix at each stage; buy when low and sell when high. This is like covered calls which is the opposite of portfolio insurance.

- Consider fixed mix strategies A (64-36 stock bond mix) and B (46-54 stock bond mix).
- The optimal stochastic programming strategy dominates
Optimal stochastic strategy vs. fixed-mix strategy
Example portfolios

A. Initial Portfolios of the Three Strategies

B. Contingent Allocations at Year 1
More evidence regarding the performance of stochastic dynamic versus fixed mix models

- A further study of the performance of stochastic dynamic and fixed mix portfolio models was made by Fleten, Hoyland and Wallace (2002).
- They compared two alternative versions of a portfolio model for the Norwegian life insurance company Gjensidige NOR, namely multistage stochastic linear programming and the fixed mix constant rebalancing study.
- They found that the multiperiod stochastic programming model dominated the fixed mix approach but the degree of dominance is much smaller out-of-sample than in-sample.
- This is because out-of-sample the random input data is structurally different from in-sample, so the stochastic programming model loses its advantage in optimally adapting to the information available in the scenario tree.
- Also the performance of the fixed mix approach improves because the asset mix is updated at each stage.
Advantages of stochastic programming over fixed-mix

**A. In-Sample Results**

Expected Maximum Tree Point

**B. Out-of-Sample Results**

Expected Maximum Tree Point

Source: Based on data from Fleten, Høyland, and Wallace.
The Russell-Yasuda Kasai Model

- Russell-Yasuda Kasai was the first large scale multiperiod stochastic programming model implemented for a major financial institution, see Henriques (1991).
- As a consultant to the Frank Russell Company during 1989-91, I designed the model. The team of David Carino, Taka Eguchi, David Myers, Celine Stacy and Mike Sylvanus at Russell in Tacoma, Washington implemented the model for the Yasuda Fire and Marine Insurance Co., Ltd in Tokyo under the direction of research head Andy Turner.
- Roger Wets and Chanaka Edirishinghe helped as consultants in Tacoma, and Kats Sawaki was a consultant to Yasuda Kasai in Japan to advise them on our work.
- Kats, a member of my 1974 UBC class in stochastic programming where we started to work on ALM models, was then a professor at Nanzan University in Nagoya and acted independently of our Tacoma group.
- Kouji Watanabe headed the group in Tokyo which included Y. Tayama, Y. Yazawa, Y. Ohtani, T. Amaki, I. Harada, M. Harima, T. Morozumi and N. Ueda.
Back in 1990/91 computations were a major focus of concern. We had a pretty good idea how to formulate the model, which was an outgrowth of the Kusy and Ziemba (1986) model for the Vancouver Savings and Credit Union and the 1982 Kallberg, White and Ziemba paper. David Carino did much of the formulation details. Originally we had ten periods and 2048 scenarios. It was too big to solve at that time and became an intellectual challenge for the stochastic programming community. Bob Entriksen, D. Jensen, R. Clark and Alan King of IBM Research worked on its solution but never quite cracked it. We quickly realized that ten periods made the model far too difficult to solve and also too cumbersome to collect the data and interpret the results and the 2048 scenarios were at that time a large number to deal with. About two years later Hercules Vladimirou, working with Alan King at IBM Research was able to effectively solve the original model using parallel processing on several workstations.
The Russell-Yasuda model was designed to satisfy the following need as articulated by Kunihiko Sasamoto, director and deputy president of Yasuda Kasai.

*The liability structure of the property and casualty insurance business has become very complex, and the insurance industry has various restrictions in terms of asset management. We concluded that existing models, such as Markowitz mean variance, would not function well and that we needed to develop a new asset/liability management model.*

*The Russell-Yasuda Kasai model is now at the core of all asset/liability work for the firm. We can define our risks in concrete terms, rather than through an abstract, in business terms, measure like standard deviation. The model has provided an important side benefit by pushing the technology and efficiency of other models in Yasuda forward to complement it. The model has assisted Yasuda in determining when and how human judgment is best used in the asset/liability process.*

From Carino et al (1994)

The model was a big success and of great interest both in the academic and institutional investment asset-liability communities.
The Yasuda Fire and Marine Insurance Company

• called Yasuda Kasai meaning fire is based in Tokyo.
• It began operations in 1888 and was the second largest Japanese property and casualty insurer and seventh largest in the world by revenue.
• It's main business was voluntary automobile (43.0%), personal accident (14.4%), compulsory automobile (13.7%), fire and allied (14.4%), and other (14.5%).
• The firm had assets of 3.47 trillion yen (US$26.2 billion) at the end of fiscal 1991 (March 31, 1992).
• In 1988, Yasuda Kasai and Russell signed an agreement to deliver a dynamic stochastic asset allocation model by April 1, 1991.
• Work began in September 1989.
• The goal was to implement a model of Yasuda Kasai's financial planning process to improve their investment and liability payment decisions and their overall risk management.

The business goals were to:
1. maximize long run expected wealth;
2. pay enough on the insurance policies to be competitive in current yield;
3. maintain adequate current and future reserves and cash levels, and
4. meet regulatory requirements especially with the increasing number of saving-oriented policies being sold that were generating new types of liabilities.
Convex piecewise linear risk measure

Note: $S = \text{slope.}$
The model needed to have more realistic definitions of operational risks and business constraints than the return variance used in previous mean-variance models used at Yasuda Kasai.

The implemented model determines an optimal multiperiod investment strategy that enables decision makers to define risks in tangible operational terms such as cash shortfalls.

The risk measure used is convex and penalizes target violations, more and more as the violations of various kinds and in various periods increase.

The objective is to maximize the discounted expected wealth at the horizon net of expected discounted penalty costs incurred during the five periods of the model.

This objective is similar to a mean variance model except it is over five periods and only counts downside risk through target violations.

I greatly prefer this approach to VaR or CVAR and its variants for ALM applications because for most people and organizations, the non-attainment of goals is more and more damaging not linear in the non-attainment (as in CVAR) or not considering the size of the non-attainment at all (as in VaR).

A reference on VaR and C-Var as risk measures is Artzner et al (1999).

Krokhma, Uryasev and Zrazhevsky (2005) apply these measures to hedge fund performance.

My risk measure is coherent.
Axiomatic development of convex risk measures - Read only if this interests you

Modified risk measures and acceptance sets, Rockafellar and Ziemba (July 2000)

The notation in what follows is basically that of the paper of Artzner, Delbaen, Eber and Heath, but the axioms are less restrictive.

**Definition 1** (modified acceptance sets). A set $\mathcal{A}$ is an acceptance set if it satisfies:

(A1) $\mathcal{A} \supset L_+.$

(A2) $\mathcal{A} \cap L_- = \emptyset.$

(A3) $\mathcal{A}$ is convex.

(A4) $\mathcal{A}$ is closed.

As a consequence of (A1)+(A3), if $X \in \mathcal{A}$ and $Y \geq X$, then $Y \in \mathcal{A}$. Axiom (A4) could be avoided, but closure would eventually come in anyway, so it seems conceptually simpler and cleaner just to impose it from the start.


Convex risk measures

Definition 2 (modified risk measures). A function $\rho$ is a risk measure if it satisfies:

\begin{align*}
\text{(R1)} & \quad \rho(X + \alpha \cdot r) = \rho(X) - \alpha. \\
\text{(R2)} & \quad \rho(\lambda_1 X_1 + \lambda_2 X_2) \leq \lambda_1 \rho(X_1) + \lambda_2 \rho(X_2) \text{ for } \lambda_1 > 0, \lambda_2 > 0, \lambda_1 + \lambda_2 = 1. \\
\text{(R3)} & \quad X \leq Y \text{ implies } \rho(X) \geq \rho(Y). \\
\text{(R4)} & \quad X \leq 0, X \neq 0, \text{ implies } \rho(X) > 0. \\
\text{(R5)} & \quad \rho(0) = 0.
\end{align*}

The combination of (R2) and (R5) replaces the more restrictive assumptions of sub-additivity and positive homogeneity of $\rho$. Axiom (R2) is of course just the convexity of $\rho$. From (R2)+(R5) we get that

\begin{align*}
\rho(\lambda X) & \leq \lambda \rho(X) \text{ when } 0 < \lambda < 1, \\
\rho(\lambda X) & \geq \lambda \rho(X) \text{ when } 1 < \lambda < \infty.
\end{align*}

These relations can be regarded as a partial substitute for positive homogeneity.
Acceptance sets and risk measures are in one-to-one correspondence

**Theorem 1** (equivalence). A one-to-one correspondence between acceptance sets $\mathcal{A}$ in the sense of Definition 1 and risk measures $\rho$ in the sense of Definition 2 is given by the reciprocal relations

$$(\mathcal{A} \to \rho) \quad \rho(X) = \min \{\alpha \mid X + \alpha \cdot r \in A\},$$

$$(\rho \to \mathcal{A}) \quad \mathcal{A} = \{X \mid \rho(X) \leq 0\}.$$ 

In this theorem, $r$ denotes the total return from a reference instrument that is fixed throughout; it is assumed that this total return is positive, no matter what. Note that the theorem asserts a one-to-one correspondence regardless of the choice of $r$, but the particular correspondence will depend on the particular choice.

Consider now all the various probability measures $P$ on the finite set $\Omega$ of future states (as specified in the particular model). Such probability measures can be viewed as generalized, or “mixed,” scenarios just as in elementary game theory mixed strategies are introduced to generalize pure strategies.
Generalized scenarios

Theorem 2 (representation by generalized scenarios). A function $\rho$ is a risk measure in the sense of Definition 1 if and only if it can be expressed in the form

$$\rho(X) = \sup \left\{ E_P[-X/r] - \varphi(P) \mid P \in \mathcal{P} \right\}$$

for some set $\mathcal{P}$ of probability measures on $\Omega$ and some function $\varphi$ on $\mathcal{P}$ such that

$$\inf \left\{ \varphi(P) \mid P \in \mathcal{P} \right\} = 0.$$  \hspace{1cm} (2)

The interpretation of $\varphi(P)$ is that it gives an upper bound to the expected “loss ratio” that can be incurred under the generalized scenario $P$ with respect to risks that are deemed acceptable, where “loss ratio” refers to loss relativized to the total return under the reference instrument $r$ (as a function of each future state) in contrast to the true loss. The difference being maximized in the expectation in (1) is thus the excess of the loss ratio $-X/r$ for the given risk $X$ with respect to $P$, over (the bound on) what might happen anyway with respect to $P$. 
Indeed, if we start from an acceptance set $\mathcal{A}$ and pass to the risk measure $\rho$ that corresponds to it as described in Theorem 1, we can get a specific representation of the type in Theorem 2 as follows. For any generalized scenario $P$ let

$$\varphi(P) = \sup \{E_P[-X/r] \mid X \in \mathcal{A}\},$$

and take $\mathcal{P}$ to consist of all $P$ such that $\varphi(P) < \infty$. Then (1) and (2) hold. Moreover, this is the canonical representation, in the sense that any other $\varphi$ that works in (1) and (2) had this $\varphi$ as its “closed convex hull.”
Model constraints and results

• The model formulates and meets the complex set of regulations imposed by Japanese insurance laws and practices.

• The most important of the intermediate horizon commitments is the need to produce income sufficiently high to pay the required annual interest in the savings type insurance policies without sacrificing the goal of maximizing long run expected wealth.

• During the first two years of use, fiscal 1991 and 1992, the investment strategy recommended by the model yielded a superior income return of 42 basis points (US$79 million) over what a mean-variance model would have produced. Simulation tests also show the superiority of the stochastic programming scenario based model over a mean variance approach.

• In addition to the revenue gains, there are considerable organizational and informational benefits.

• The model had 256 scenarios over four periods plus a fifth end effects period.

• The model is flexible regarding the time horizon and length of decision periods, which are multiples of quarters.

• A typical application has initialization, plus period 1 to the end of the first quarter, period 2 the remainder of fiscal year 1, period 3 the entire fiscal year 2, period 4 fiscal years 3, 4, and 5 and period 5, the end effects years 6 on to forever.
Multistage stochastic linear programming structure of the Russell-Yasuda Kasai model

Note: $C = \text{cost coefficient.}$
The Russell-Yasuda Kasai model

The basic Russell-Yasuda Kasai model has the following elements; see Cariño and Ziemba (1998) for additional details. This simplified formulation does not include additional types of shortfalls, indirect investments (tokkin funds and foreign subsidiaries), regulatory restrictions, multiple accounts, loan assets, the effects of taxes and end effects.

The stages are $t = 0, \ldots, T$.

Decision variables are:

\[
\begin{align*}
V_t &= \text{total fund market value at } t, \\
x_{nt} &= \text{market value in asset } n \text{ at } t, \\
w_{t+1} &= \text{income shortfall at } t+1, \text{ and} \\
v_{t+1} &= \text{income surplus at } t+1.
\end{align*}
\]

Random variables are:

\[
\begin{align*}
RP_{n,t+1} &= \text{price return of asset } n \text{ from end of } t \text{ to end of } t_1 \\
RI_{n,t+1} &= \text{income return of asset } n \text{ from end of } t \text{ to end of } t + 1
\end{align*}
\]

Random variables appearing in the right-hand side are:

\[
\begin{align*}
E_{t+1} &= \text{deposit inflow from end of } t \text{ to end of } t + 1 \\
P_{t+1} &= \text{principal payout from end of } t \text{ to end of } t + 1 \\
I_{t+1} &= \text{interest payout from end of } t \text{ to end of } t + 1 \\
g_{t+1} &= \text{rate at which interest is credited to policies from end of } t \text{ to end of } t + 1 \\
L_t &= \text{liability valuation at from } t.
\end{align*}
\]

The piecewise linear convex shortfall risk measure is $c_t(\cdot)$.
The objective of the model is to allocate discounted fund value among asset classes to maximize the expected wealth at the end of the planning horizon $T$ less expected penalized shortfalls accumulated throughout the planning horizon.

$$\text{Maximize } E \left[ V_T - \sum_{t=1}^{T} c_t(w_t) \right]$$

subject to budget constraints

$$\sum_{n} X_{nt} - V_t = 0,$$

asset accumulation relations

$$V_{t+1} - \sum_{n} (1 + RP_{nt+1} + RI_{nt+1})X_{nt} = F_{t+1} - P_{t+1} - I_{t+1},$$

income shortfall constraints

$$\sum_{n} RI_{nt+1}X_{nt} + w_{t+1} - v_{t+1} = g_{t+1}L_t,$$

and nonnegativity constraints

$$X_{nt} \geq 0, \quad v_{t+1} \geq 0, \quad w_{t+1} \geq 0,$$

for $t = 0, 1, 2, \ldots, T - 1$. Liability balances and cash flows are computed to satisfy the liability accumulation relations

$$L_{t+1} = (1 + g_{t+1})L_t + F_{t+1} - P_{t+1} - I_{t+1}, \quad t = 0, \ldots, T - 1.$$
Stochastic linear programs are giant linear programs

The model has the base linear program which has a single scenario in the multiperiod deterministic formulation. Then the random variable scenarios are generated with a tree specification. Then combining the base linear program and the tree, gives the model as a very large extensive form linear program.
The dimensions of the implemented problem:

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<th>Scen</th>
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Yasuda Kasai’s asset/liability decision-making process

Note: UB = upper bound; LB = lower bound; CG = company growth.
Yasuda Fire and Marine faced the following situation:

1. an increasing number of savings-oriented policies were being sold which had new types of liabilities
2. the Japanese Ministry of Finance imposed many restrictions through insurance law and that led to complex constraints
3. the firm's goals included both current yield and long-run total return and that lead to risks and objectives were multidimensional

- The insurance policies were complex with a part being actual insurance and another part an investment with a fixed guaranteed amount plus a bonus dependent on general business conditions in the industry.
- The insurance contracts are of varying length; maturing, being renewed or starting in various time periods, and subject to random returns on assets managed, insurance claims paid, and bonus payments made.
- The insurance company's balance sheet is as follows with various special savings accounts
- There are many regulations on assets including restrictions on equity, loans, real estate, foreign investment by account, foreign subsidiaries and tokkin (pooled accounts).
Asset classes for the Russell-Yasuda Kasai model

<table>
<thead>
<tr>
<th>Asset</th>
<th>Associated Index</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cash bonds</td>
<td>Nomura Bond Performance Index</td>
</tr>
<tr>
<td>Convertible bonds</td>
<td>Nikko Research Convertible Bond Index</td>
</tr>
<tr>
<td>Domestic equities</td>
<td>Tokyo Stock Price Index (TOPIX)</td>
</tr>
<tr>
<td>Hedged foreign bonds</td>
<td>Salomon Brothers World Bond Index (or hedged equivalent)</td>
</tr>
<tr>
<td>Hedged foreign equities</td>
<td>Morgan Stanley World Equity Index (or hedged equivalent)</td>
</tr>
<tr>
<td>Unhedged foreign bonds</td>
<td>Salomon Brothers World Bond Index</td>
</tr>
<tr>
<td>Unhedged foreign equities</td>
<td>Morgan Stanley World Equity Index</td>
</tr>
<tr>
<td>Loans</td>
<td>Average lending rates (trust/long-term credit or long-term prime rates)</td>
</tr>
<tr>
<td>Money trusts and so on</td>
<td>Call rates (overnight with collateral)</td>
</tr>
</tbody>
</table>

*Note:* Life insurance company general accounts are an asset class but have no associated index.
Expected allocations in the initialization period (INI)

<table>
<thead>
<tr>
<th>Total (100 million)</th>
<th>Percentage</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cash</td>
<td>2,053</td>
</tr>
<tr>
<td>Loans (floating rate)</td>
<td>5,598</td>
</tr>
<tr>
<td>Loans (fixed rate)</td>
<td>5,674</td>
</tr>
<tr>
<td>Bonds</td>
<td>2,898</td>
</tr>
<tr>
<td>Equity</td>
<td>1,426</td>
</tr>
<tr>
<td>Foreign bonds</td>
<td>3,277</td>
</tr>
<tr>
<td>Foreign equity</td>
<td>875</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td><strong>21,800</strong></td>
</tr>
</tbody>
</table>

*Note: Total book value 1 = 22,510 (¥100 million). Total book value 2 = 34,875 (¥100 million).*
Expected allocations in the end-effects period

(¥ 100 million)

<table>
<thead>
<tr>
<th></th>
<th>General</th>
<th>Savings</th>
<th>Special Savings 1</th>
<th>Special Savings 2</th>
<th>Exogenous</th>
<th>Total</th>
<th>Percentage</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cash</td>
<td>0</td>
<td>44</td>
<td>0</td>
<td>36</td>
<td>0</td>
<td>80</td>
<td>0.1%</td>
</tr>
<tr>
<td>Bonds</td>
<td>5,945</td>
<td>17</td>
<td>14,846</td>
<td>1,311</td>
<td>0</td>
<td>22,119</td>
<td>40.1%</td>
</tr>
<tr>
<td>Equity</td>
<td>0</td>
<td>0</td>
<td>4</td>
<td>0</td>
<td>18,588</td>
<td>18,592</td>
<td>33.7%</td>
</tr>
<tr>
<td>Foreign bonds</td>
<td>2,837</td>
<td>1,094</td>
<td>0</td>
<td>0</td>
<td>18,588</td>
<td>3,931</td>
<td>7.1%</td>
</tr>
<tr>
<td>Foreign equity</td>
<td>0</td>
<td>4,650</td>
<td>6,022</td>
<td>562</td>
<td>0</td>
<td>11,234</td>
<td>20.4%</td>
</tr>
<tr>
<td>Total</td>
<td>8,782</td>
<td>5,804</td>
<td>20,072</td>
<td>1,908</td>
<td>18,588</td>
<td>55,154</td>
<td></td>
</tr>
</tbody>
</table>

*Note: Total book value 1 = 28,566. Total book value 2 = 50,547.*
In summary

1. The 1991 Russell Yasuda Kasai Model was then the largest application of stochastic programming in financial services.

2. There was a significant ongoing contribution to Yasuda Kasai’s financial performance US$79 million and US$9 million in income and total return, respectively, over FY91-92 and it has been in use since then.

3. The basic structure is portable to other applications because of flexible model generation.

4. A substantial potential impact in performance of financial services companies.

5. The top 200 insurers worldwide have in excess of $10 trillion in assets.

6. Worldwide pension assets are also about $7.5 trillion, with a $2.5 trillion deficit.

7. The industry is also moving towards more complex products and liabilities and risk based capital requirements.
“Most people still spend more time planning for their vacation than for their retirement”

Citigroup

“Half of the investors who hold company stock in their retirement accounts thought it carried the same or less risk than money market funds”

Boston Research Group
The Pension Fund Situation

Stock market declines can be very hard on pension funds in several ways:
• For defined benefit plans there can be shortfalls to the fund manager and sometimes pension plans go bankrupt
• For defined contribution plans, payouts and balances fall and this creates image and employee morale problems

Some examples:
UK professors fund, Ford, GM n the US in shortfall
The Pension Fund Situation in Europe

• Rapid ageing of the developed world’s populations - the retiree group, those 65 and older, will roughly double from about 20% to about 40% of compared to the worker group, those 15-64

• Better living conditions, more effective medical systems, a decline in fertility rates and low immigration into the Western world contribute to this ageing phenomenon.

• By 2030 two workers will have to support each pensioner compared with four now.

• Contribution rates will rise

• Rules to make pensions less desirable will be made
  • raise retirement age, lower benefits
## Asset structure of European Pension Funds in Percent, 1997

<table>
<thead>
<tr>
<th>Countries</th>
<th>Equity</th>
<th>Fixed Income</th>
<th>Real Estate</th>
<th>Cash &amp; STP</th>
<th>Other</th>
</tr>
</thead>
<tbody>
<tr>
<td>Austria</td>
<td>4.1</td>
<td>82.4</td>
<td>1.8</td>
<td>1.6</td>
<td>10.0</td>
</tr>
<tr>
<td>Denmark</td>
<td>23.2</td>
<td>58.6</td>
<td>5.3</td>
<td>1.8</td>
<td>11.1</td>
</tr>
<tr>
<td>Finland</td>
<td>13.8</td>
<td>55.0</td>
<td>13.0</td>
<td>18.2</td>
<td>0.0</td>
</tr>
<tr>
<td>France</td>
<td>12.6</td>
<td>43.1</td>
<td>7.9</td>
<td>6.5</td>
<td>29.9</td>
</tr>
<tr>
<td>Germany</td>
<td>9.0</td>
<td>75.0</td>
<td>13.0</td>
<td>3.0</td>
<td>0.0</td>
</tr>
<tr>
<td>Greece</td>
<td>7.0</td>
<td>62.9</td>
<td>8.3</td>
<td>21.8</td>
<td>0.0</td>
</tr>
<tr>
<td>Ireland</td>
<td>58.6</td>
<td>27.1</td>
<td>6.0</td>
<td>8.0</td>
<td>0.4</td>
</tr>
<tr>
<td>Italy</td>
<td>4.8</td>
<td>76.4</td>
<td>16.7</td>
<td>2.0</td>
<td>0.0</td>
</tr>
<tr>
<td>Netherlands</td>
<td>36.8</td>
<td>51.3</td>
<td>5.2</td>
<td>1.5</td>
<td>5.2</td>
</tr>
<tr>
<td>Portugal</td>
<td>28.1</td>
<td>55.8</td>
<td>4.6</td>
<td>8.8</td>
<td>2.7</td>
</tr>
<tr>
<td>Spain</td>
<td>11.3</td>
<td>60.0</td>
<td>3.7</td>
<td>11.5</td>
<td>13.5</td>
</tr>
<tr>
<td>Sweden</td>
<td>40.3</td>
<td>53.5</td>
<td>5.4</td>
<td>0.8</td>
<td>0.1</td>
</tr>
<tr>
<td>U.K.</td>
<td>72.9</td>
<td>15.1</td>
<td>5.0</td>
<td>7.0</td>
<td>0.0</td>
</tr>
<tr>
<td><strong>Total EU</strong></td>
<td><strong>53.6</strong></td>
<td><strong>32.8</strong></td>
<td><strong>5.8</strong></td>
<td><strong>5.2</strong></td>
<td><strong>2.7</strong></td>
</tr>
<tr>
<td>US*</td>
<td>52</td>
<td>36</td>
<td>4</td>
<td>8</td>
<td>n.a.</td>
</tr>
<tr>
<td>Japan*</td>
<td>29</td>
<td>63</td>
<td>3</td>
<td>5</td>
<td>n.a.</td>
</tr>
</tbody>
</table>

There have been three periods in the US markets where equities had essentially had essentially zero gains in nominal terms, 1899 to 1919, 1929 to 1954 and 1964 to 1981.

The trend is up but it's quite bumpy.
Many asset classes. Goods you will use and services you would like such as doctors are important for pensioners.

“Actually, with pension-fund mismanagement and Social Security in doubt, burying bones is once again an attractive retirement option.”
What is InnoALM?

- A multi-period stochastic linear programming model designed by Ziemba and implemented by Geyer with input from Herold and Kontriner
- For Innovest to use for Austrian pension funds
- A tool to analyze Tier 2 pension fund investment decisions

Why was it developed?

- To respond to the growing worldwide challenges of ageing populations and increased number of pensioners who put pressure on government services such as health care and Tier 1 national pensions
- To keep Innovest competitive in their high level fund management activities
Features of InnoALM

• A multiperiod stochastic linear programming framework with a flexible number of time periods of varying length.
• Generation and aggregation of multiperiod discrete probability scenarios for random return and other parameters
• Various forecasting models
• Scenario dependent correlations across asset classes
• Multiple co-variance matrices corresponding to differing market conditions
• Constraints reflect Austrian pension law and policy
Technical features include

• Concave risk averse preference function maximizes expected present value of terminal wealth net of expected convex (piecewise linear) penalty costs for wealth and benchmark targets in each decision period.

• InnoALM user interface allows for visualization of key model outputs, the effect of input changes, growing pension benefits from increased deterministic wealth target violations, stochastic benchmark targets, security reserves, policy changes, etc.

• Solution process using the IBM OSL stochastic programming code is fast enough to generate virtually online decisions and results and allows for easy interaction of the user with the model to improve pension fund performance.

InnoALM reacts to all market conditions: severe as well as normal

The scenarios are intended to anticipate the impact of various events, even if they have never occurred before
Asset Growth

Deterministic

achieved

required

T

maturity

surplus is desirable

Stochastic

shortfalls

0

deposit

T

maturity
Objective: Max $E_S[\text{discounted } W_T] – R_A[\text{discounted sum of policy target violations of type I in period } t, \text{ over periods } t=1, \ldots, T]$

Penalty cost convex

Concave risk averse

$R_A = \text{risk aversion index}$  
2 risk taker
4 pension funds
8 conservative

See Kallberg and Ziemba, Management Science, 1983 for the theory
Description of the Pension Fund

Siemens AG Österreich is the largest privately owned industrial company in Austria. Turnover (EUR 2.4 Bn. in 1999) is generated in a wide range of business lines including information and communication networks, information and communication products, business services, energy and traveling technology, and medical equipment.

• The Siemens Pension fund, established in 1998, is the largest corporate pension plan in Austria and follows the defined contribution principle.

• More than 15,000 employees and 5,000 pensioners are members of the pension plan with about EUR 500 million in assets under management.

• Innovest Finanzdienstleistungs AG, which was founded in 1998, acts as the investment manager for the Siemens AG Österreich, the Siemens Pension Plan as well as for other institutional investors in Austria.

• With EUR 2.2 billion in assets under management, Innovest focuses on asset management for institutional money and pension funds.

• The fund was rated the 1st of 19 pension funds in Austria for the two-year 1999/2000 period
Factors that led Innovest to develop the pension fund asset-liability management model InnoALM

- Changing demographics in Austria, Europe and the rest of the globe, are creating a higher ratio of retirees to working population.

- Growing financial burden on the government making it paramount that private employee pension plans be managed in the best possible way using systematic asset-liability management models as a tool in the decision making process.

- A myriad of uncertainties, possible future economic scenarios, stock, bond and other investments, transactions costs and liquidity, currency aspects, liability commitments

- Both Austrian pension fund law and company policy suggest that multiperiod stochastic linear programming is a good way to model these uncertainties
Factors that led to the development of InnoALM, cont’d

• Faster computers have been a major factor in the development and use of such models, SP problems with millions of variables have been solved by my students Edirisinghe and Gassmann and by many others such as Dempster, Gonzio, Kouwenberg, Mulvey, Zenios, etc

• Good user friendly models now need to be developed that well represent the situation at hand and provide the essential information required quickly to those who need to make sound pension fund asset-liability decisions.

InnoALM and other such models allow pension funds to strategically plan and diversify their asset holdings across the world, keeping track of the various aspects relevant to the prudent operation of a company pension plan that is intended to provide retired employees a supplement to their government pensions.
InnoALM Project Team

- For the Russell Yasuda-Kasai models, we had a very large team and overhead costs were very high.
- At Innovest we were a team of four with Geyer implementing my ideas with Herold and Kontriner contributing guidance and information about the Austrian situation.
- The IBM OSL Stochastic Programming Code of Alan King was used with various interfaces allowing lower development costs [for a survey of codes see in Wallace-Ziemba, 2005, *Applications of Stochastic Programming*, a friendly users guide to SP modeling, computations and applications, SIAM MPS]

The success of InnoALM demonstrates that a small team of researchers with a limited budget can quickly produce a valuable modeling system that can easily be operated by non-stochastic programming specialists on a single PC
Innovest InnoALM model

Deterministic wealth targets grow 7.5% per year

Stochastic benchmark targets on asset returns

\[ \tilde{R}_B B + \tilde{R}_S S + \tilde{R}_C C + \tilde{R}_{RE} RE + M_{it} \geq \]

\[ \tilde{R}_{BBM} BBM + \tilde{R}_{SBM} SBM + \tilde{R}_{CBM} CBM + \tilde{R}_{REBM} REBM \]

Stochastic benchmark returns with asset weights B, S, C, RE, \( M_{it} \)=shortfall to be penalized
Examples of national investment restrictions on pension plans

<table>
<thead>
<tr>
<th>Country</th>
<th>Investment Restrictions</th>
</tr>
</thead>
<tbody>
<tr>
<td>Germany</td>
<td>Max. 30% equities, max. 5% foreign bonds</td>
</tr>
<tr>
<td>Austria</td>
<td>Max. 40% equities, max. 45% foreign securities, min. 40% EURO bonds, 5% options</td>
</tr>
<tr>
<td>France</td>
<td>Min. 50% EURO bonds</td>
</tr>
<tr>
<td>Portugal</td>
<td>Max. 35% equities</td>
</tr>
<tr>
<td>Sweden</td>
<td>Max. 25% equities</td>
</tr>
<tr>
<td>UK, US</td>
<td>Prudent man rule</td>
</tr>
</tbody>
</table>

*Source: European Commission (1997)*

In new proposals, the limit for worldwide equities would rise to 70% versus the current average of about 35% in EU countries.

The model gives insight into the wisdom of such rules and portfolios can be structured around the risks.
Formulating the InnoALM as a multistage stochastic linear programming model

- The model determines the optimal purchases and sales for each of $N$ assets in each of $T$ planning periods.
- Typical asset classes used at Innovest are US, Pacific, European, and Emerging Market equities and US, UK, Japanese and European bonds.
- A concave risk averse utility function is to maximize expected terminal wealth less convex penalty costs subject to linear constraints.
- The convex risk measure is approximated by a piecewise linear function, so the model is a multiperiod stochastic linear program.
- The non-negative decision variables are wealth (after transactions), and purchases and sales for each asset ($i=1,\ldots,N$).
- Purchases and sales are in periods $t=0,\ldots,T-1$.
- Except for $t=0$, purchases and sales are scenario dependent.
Wealth accumulates over time for a $T$ period model according to

$$
W_{i0} = W_{i0}^{\text{init}} + P_{i0} - S_{i0}, \quad t=0
$$

$$
\tilde{W}_{i1} = \tilde{R}_{i1} W_{i0} + \tilde{P}_{i1} - \tilde{S}_{i1}, \quad t=1
$$

$$
\tilde{W}_{it} = \tilde{R}_{it} \tilde{W}_{i,t-1} + \tilde{P}_{it} - \tilde{S}_{it}, \quad t=2,\ldots,T-1, \text{ and}
$$

$$
\tilde{W}_{iT} = \tilde{R}_{iT} \tilde{W}_{i,T-1}, \quad t=T.
$$

$W_{i0}^{\text{init}}$ is the prespecified initial value of asset $i$.

- There is no uncertainty in the initialization period $t=0$.
- Tildas denote scenario-dependent random parameters or decision variables.
- Returns are associated with time intervals. $\tilde{R}_{it}$ ($t=1,\ldots,T$) are the (random) gross returns for asset $i$ between $t-1$ and $t$. 


The budget constraints are

\[
\sum_{i=1}^{N} P_{i0} (1 + tcp_{i}) = \sum_{i=1}^{N} S_{i0} (1 - tcs_{i}) + C_{0} \quad t=0, \text{ and}
\]

\[
\sum_{i=1}^{N} \tilde{P}_{i} (1 + tcp_{i}) = \sum_{i=1}^{N} \tilde{S}_{i} (1 - tcs_{i}) + C_{t} \quad t=1, \ldots, T-1,
\]

where \( tcp_{i} \) and \( tcs_{i} \) denote asset-specific linear transaction-costs for purchases and sales, and \( C_{t} \) is the fixed (non-random) net cashflow (inflow if positive).
Portfolio weights can be constrained over linear combinations (subsets) of assets or individual assets via

$$\sum_{i=1}^{N} \bar{W}_i - \theta_U \sum_{i=1}^{N} \bar{W}_i \leq 0,$$  \quad \text{and}

$$-\sum_{i=1}^{N} \bar{W}_i + \theta_L \sum_{i=1}^{N} \bar{W}_i \leq 0, \quad t=1,\ldots,T-1$$

where $\theta_U$ is the maximum percentage and $\theta_L$ is the minimum percentage of the subsets $U$ and $L$ of assets $i=1,\ldots,N$ included in the restrictions.

- The $\theta_U$'s, $\theta_L$'s, $U$'s and $L$'s may be time dependent.

- Risk is measured as a weighted discounted convex function of target violation shortfalls of various types in various periods. In a typical application, the deterministic wealth target $\bar{W}_i$ is assumed to grow by 7.5% in each year.

- The 7.5% accounts for inflation plus growth of the number of retirees.
The wealth targets are modeled via

$$\sum_{i} (\tilde{W}_i - \tilde{P}_i + \tilde{S}_i) + \tilde{M}_i^w \geq \tilde{W}_i \quad t=1,...,T,$$

- where $\tilde{M}_i^w$ are non-negative wealth-target shortfall variables.
- The shortfall is penalized using a piecewise linear convex risk measure using the variables and constraints

$$\tilde{M}_i^w = \sum_{j=1}^{m} \tilde{M}_j^w, \quad t=1,...,T$$

$$\tilde{M}_j^w \leq b_j - b_{j-1}, \quad t=1,...,T; j=1,...,m-1,$$

- where $\tilde{M}_j^w$ is the wealth target shortfall associated with segment $j$ of the cost-function,
- $b_j$ is the $j$-th breakpoint of the risk measure function ($b_0=0$), and $m$ is the number of segments of the function.
- A piecewise linear approximation to the convex quadratic risk measure is used so the model remains linear.
The appropriateness of the quadratic function is discussed below. Convexity guarantees that if $\tilde{M}^{w}_{j} > 0$ then $\tilde{M}^{w}_{j-1,j}$ is at its maximum and if $\tilde{M}^{w}_{j}$ is not at its maximum then $\tilde{M}^{w}_{j-1,j} = 0$.

Stochastic benchmark goals can also be set by the user and are similarly penalized for underachievement. The benchmark target $\tilde{B}$, is scenario dependent. It is based on stochastic asset returns and fixed asset weights $\alpha$, defining the benchmark portfolio

$$\tilde{B}_t = W_0 \sum_{j=1}^{t} \sum_{i=1}^{N} \alpha_i \tilde{R}_{ij}.$$  

The corresponding shortfall constraints are

$$\sum_{i=1}^{N} (\tilde{W}_i - \tilde{P}_i + \tilde{S}_i) + \tilde{M}^{\beta}_{t} \geq \tilde{B}, t=1,\ldots,T,$$

where $\tilde{M}^{\beta}_{t}$ is the benchmark-target shortfall. These shortfalls are also penalized with a piecewise linear convex risk measure.
• If total wealth exceeds the target, a fraction $\gamma = 10\%$ of the exceeding amount is allocated to a reserve account and invested in the same way as other available funds.

• However, the wealth targets at future stages are adjusted.

• Additional non-negative decision variables $\bar{D}_t$ are introduced and the wealth target constraints become

$$\sum_{i=1}^{N}(\bar{W}_i - \bar{P}_i + \bar{S}_i) - \bar{D}_t + \bar{M}^w = \bar{W}_i + \sum_{j=1}^{t-1} \gamma \bar{D}_{t-j}, \quad t=1,\ldots,T-1, \text{ where } \bar{D}_1 = 0.$$ 

• Since pension payments are based on wealth levels, increasing these levels increases pension payments.

• The reserves provide security for the pension plan’s increase of pension payments at each future stage.
The pension plan's objective function is to maximize the expected discounted value of terminal wealth in period $T$ net of the expected discounted penalty costs over the horizon from the convex risk measures $c_s(\cdot)$ for the wealth- and benchmark-targets, respectively,

$$\text{Max } E \left[ d_T \sum_{s=1}^{N} \bar{W}_{ts} - \lambda \sum_{s=1}^{T} d_s w_s \left( \sum_{k \in (s, A)} v_k c^*_s (\bar{M}^s_k) \right) \right].$$

- Expectation is over $T$ period scenarios $S_T$.
- The discount factors $d_s$ are related to the interest rate $r$ by $d_s = (1 + r)^{-s}$.
- Usually $r$ is taken to be the three or six month Treasury-bill rate.
- The $v_k$ are weights for the wealth- and benchmark-shortfalls and the $w_s$ are weights for the weighted sum of shortfalls at each stage normalized via

$$\sum_{k \in (s, A)} v_k = 1 \quad \text{and} \quad \sum_{s=1}^{T} w_s = T.$$
• In the implementation of the model the penalty function $c_k(M^k)$ corresponds to a quadratic utility function.

• Kallberg and Ziemba (1983) show for normally distributed asset returns that varying the average Arrow-Pratt absolute risk aversion index $R_A$ traces out the whole spectrum of risk attitudes of all concave utility functions.

• The most aggressive behavior is log utility which has $R_A = 1/$wealth which is essentially zero.

• Typical 60-40 stock-bond pension funds have $R_A = 4$.

• The Kallberg-Ziemba (1983) results indicate that for computational purposes the quadratic utility function $u(w) = w - R_A/2w^2$ will suffice and is easier to use in the optimization.

• The error in this approximation is close to zero and well below the accuracy of the data.
• The parameter $\lambda$ in the objective corresponds to $R_d/2$ which in the quadratic utility function is the weight assigned to risk measured in terms of variance.
• The objective function of the InnoALM model only penalizes wealth and benchmark target shortfalls.
• If the target growth is roughly equal to the average return of the portfolio, shortfalls measure only negative deviations from the mean, whereas variance is based on positive and negative deviations.
• This implies that shortfalls only account for about half of the variance.
• Therefore, to obtain results in agreement with a quadratic utility function, we use $\lambda = R_d$, rather than $R_d/2$, in the objective function.
• To obtain a solution to the allocation problem for general levels of total initial wealth $w_0$, we use the rescaled parameter $\lambda = R_d / w_0$ in the objective function.
Using a quadratic function, the penalty function $c_k(M^k)$ is

$$c_k(M^k) = \sum_{j=1}^{n} \overline{M}_{jt}^k (b_{j-1} + b_j), \quad \overline{M}_{jt}^k \leq b_{j} - b_{j-1}, \quad \text{with } b_0 = 0.$$

- Uncertainty is modelled using multiperiod discrete probability scenarios using statistical properties of the assets' returns.
- A scenario tree is defined by the number of stages and the number of arcs leaving a particular node.
Implementation, output and sample results

• An Excel™ spreadsheet is the user interface.
• The spreadsheet is used to select assets, define the number of periods and the scenario node-structure.
• The user specifies the wealth targets, cash in- and out-flows and the asset weights that define the benchmark portfolio (if any).
• The input-file contains a sheet with historical data and sheets to specify expected returns, standard deviations, correlation matrices and steering parameters.
• A typical application with 10,000 scenarios takes about 7-8 minutes for simulation, generating SMPS files, solving and producing output on a 1.2 Ghz Pentium III notebook with 376 MB RAM. For some problems, execution times can be 15-20 minutes.
Elements of InnoALM

Front-end user interface (Excel)
- Periods (targets, node structure, fixed cash-flows, ...)
- Assets (selection, initial values, transaction costs, ...)
- Liability data
- Statistics (mean, standard deviation, correlation)
- Bounds
- Weights
- Historical data
- Options
- Controls

GAUSS
- read input
- compute statistics
- simulation returns (generate scenarios)
- generate SMPS files

IBMOSL solver
- read SMPS input files
- solve the problem
- generate output file (solutions for all nodes and variables)

Rear-end user interface (GAUSS)
- read optimal solutions
- generate tables and graphs
- retain key variables in memory to allow for further analyses
Example

• Four asset classes (stocks Europe, stocks US, bonds Europe, and bonds US) with five periods (six stages).
• The periods are twice 1 year, twice 2 years and 4 years (10 years in total).
• 10000 scenarios based on a 100-5-5-2-2 node structure.
• The wealth target grows at an annual rate of 7.5%.
• $R_A=4$ and the discount factor equals 5.
State dependent correlation matrices of InnoALM are a new feature, which have not yet been used in pension planning or asset allocation models.

- InnoALM uses three different correlation matrices and corresponding sets of standard deviations.
- The choice of a specific correlation matrix depends on the level of stock return volatility.
- We distinguish 'extreme' (or 'crash') periods, 'highly volatile' periods and 'normal' periods.
- Each of the three periods or regimes is assigned a probability of occurrence $p_j$ ($j=1,2,3$). Harvey (1991), Karolyi and Stulz (1996), Solnik, et al. (1996), and Das and Uppal (2004) study changing correlation structures over time.
- To estimate correlations and standard deviations for the three regimes, we use the regression approach suggested by Solnik, et al. (1996).
- Using monthly time series we compute moving average (window length 36 months) estimates of correlations among all assets and standard deviations of US equity returns. Correlations are regressed on US stock return volatilities. The estimated regression equations are used to predict correlations for the three regimes.
Scenario dependent correlations matrices

Means, standard deviations & correlations based on 1970-2000 data

<table>
<thead>
<tr>
<th>Scenario</th>
<th>Stocks Europe</th>
<th>Stocks US</th>
<th>Bonds Europe</th>
<th>Bonds US</th>
</tr>
</thead>
<tbody>
<tr>
<td>normal periods (70% of the time)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Stocks US</td>
<td>.755</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Bonds Europe</td>
<td>.334</td>
<td>.286</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Bonds US</td>
<td>.514</td>
<td>.780</td>
<td>.333</td>
<td></td>
</tr>
<tr>
<td>Standard dev</td>
<td>14.6</td>
<td>17.3</td>
<td>3.3</td>
<td>10.9</td>
</tr>
<tr>
<td>high volatility (20% of the time)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Stocks US</td>
<td>.786</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Bonds Europe</td>
<td>.171</td>
<td>.100</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Bonds US</td>
<td>.435</td>
<td>.715</td>
<td>.159</td>
<td></td>
</tr>
<tr>
<td>Standard dev</td>
<td>19.2</td>
<td>21.1</td>
<td>4.1</td>
<td>12.4</td>
</tr>
<tr>
<td>extreme periods (10% of the time)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Stocks US</td>
<td>.832</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Bonds Europe</td>
<td>-.075</td>
<td>-.182</td>
<td>-.104</td>
<td></td>
</tr>
<tr>
<td>Bonds US</td>
<td>3.15</td>
<td>6.18</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Standard dev</td>
<td>21.7</td>
<td>27.1</td>
<td>4.4</td>
<td>12.9</td>
</tr>
<tr>
<td>average period</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Stocks US</td>
<td>.769</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Bonds Europe</td>
<td>.261</td>
<td>.202</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Bonds US</td>
<td>.478</td>
<td>.751</td>
<td>.255</td>
<td></td>
</tr>
<tr>
<td>Standard dev</td>
<td>16.4</td>
<td>19.3</td>
<td>3.6</td>
<td>11.4</td>
</tr>
<tr>
<td>all periods</td>
<td>Mean</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>10.6</td>
<td>10.7</td>
<td>6.5</td>
<td>7.2</td>
</tr>
</tbody>
</table>
Deriving the scenario dependent correlations

There are three different regimes

- assume 10% of the time equity markets are extremely volatile,
- 20% of the time markets are characterized by high volatility and
- 70% of the time, markets are normal.

Each regime is defined by its median

- For *Normal Periods* the 35th percentile of US equity return volatility located at the center of the 70% 'normal' range defines 'normal' periods.
- *Highly volatile* periods are based on the 80th volatility percentile and
- *Extreme* periods are based on the 95th percentile.

The associated correlations reflect the return relationships that typically prevailed during those market conditions as in the previous table.

For example, if the 35th percentile of volatility is 0.173 (p.a.) the expected correlation between US and European stocks is $0.62 + 2.7 \cdot 0.173 / \sqrt{12} = 0.755$
The correlations show a distinct pattern across the three regimes

• Correlations among stocks tend to increase as stock return volatility rises, whereas the correlations between stocks and bonds tend to decrease.

• European Bonds may serve as a hedge for equities during extremely volatile periods since bonds and stocks returns, which are usually positively correlated, are then negatively correlated.
Regression Equations Relating Asset Correlations and US Stock Return Volatility (monthly returns; Jan 1989–Sep 2000; 141 observations)

<table>
<thead>
<tr>
<th>Correlation between</th>
<th>constant</th>
<th>slope w.r.t. US stock volatility</th>
<th>t-statistic of slope</th>
<th>$R^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Stocks Europe – Stocks US</td>
<td>0.62</td>
<td>2.7</td>
<td>6.5</td>
<td>0.23</td>
</tr>
<tr>
<td>Stocks Europe – Bonds Europe</td>
<td>1.05</td>
<td>-14.4</td>
<td>-16.9</td>
<td>0.67</td>
</tr>
<tr>
<td>Stocks Europe – Bonds US</td>
<td>0.86</td>
<td>-7.0</td>
<td>-9.7</td>
<td>0.40</td>
</tr>
<tr>
<td>Stocks US – Bonds Europe</td>
<td>1.11</td>
<td>-16.5</td>
<td>-25.2</td>
<td>0.82</td>
</tr>
<tr>
<td>Stocks US – Bonds US</td>
<td>1.07</td>
<td>-5.7</td>
<td>-11.2</td>
<td>0.48</td>
</tr>
<tr>
<td>Bonds Europe – Bonds US</td>
<td>1.10</td>
<td>-15.4</td>
<td>-12.8</td>
<td>0.54</td>
</tr>
</tbody>
</table>
Scenario Tree with a 2-2-3 Node Structure (12 Scenarios) for a 3-period problem with 4 stages.

- The tree starts with a single node which corresponds to the present state \( (t=0) \).
- Decisions are made at each node of the tree and depend on the current state which reflects previous decisions and uncertain future paths.
• A single scenario $s_t$ is a trajectory that corresponds to a unique path leading from the single node at stage 1 ($t=0$) to a single node at $t$.

• Two scenarios $s_t'$ and $s_t''$ are identical until $t-1$ (i.e. $s_{t-1}' = s_{t-1}''$) and differ in subsequent periods $t, ..., T$.

• The scenario assigns specific values to all uncertain parameters along the trajectory, i.e., asset returns and benchmark targets for all periods.

• Given all $T$ period scenarios $S_T$ and their respective probabilities, one has a complete description of the uncertainty of the model.
Correlated random returns are simulated using the following procedure.

- For each asset \( i \) we generate \( n_i \) standardized random numbers \( z_{ni} \), where \( n_i \) is the number of nodes in period \( t \).
- \( z_{ni} \) may have a normal-, \( t \)-, or historical distribution, depending on the asset and the choice of the user.
- The \( n_i \)-dimensional vectors \( z_{ni} \) are used to compile the \( n_i \times N \) matrix \( \tilde{Z} \).
- The correlation among assets is modeled by multiplying the matrix \( \tilde{Z} \) with the Cholesky decomposition of the correlation matrix \( C \): \( \tilde{Y} = \tilde{Z} \cdot \text{chol}(C) \).
- This procedure essentially uses a normal multivariate copula with normal or non-normal marginals.
Simulated gross returns for each asset are obtained by multiplying each column of $\bar{Y}$ with the standard deviation $\sigma_i$ of asset $i$ and adding the asset's mean $\mu_i$ where both are adjusted for the length $\tau_i$ of planning period $t$ via

$$\bar{R}_i = (1 + \mu_i)\bar{Y}_i + \bar{Y}_i \sigma_i \sqrt{\tau_i}.$$  

Mixing of correlations is obtained by generating three subsets of simulated returns as described, but using a different correlation matrix $C^j$ in each subset yielding three sets of returns $\bar{R}_i^j$ ($j=1,2,3$).

At any particular period the set of all nodes $n_i$ is randomly partitioned into three subsets corresponding to the three volatility regimes. We use all available nodes of a period to define the subsets (i.e. $n_i=k_1\cdot k_2\cdots k_l$, where $k_i$ is the number of paths leaving from node $i$–1).

The number of elements $n_i^j$ in each set is determined by the prespecified probability $p^j$ of the three regimes via $n_i^j = n_i p^j$ where $n_i^j$ is rounded up to the nearest integer.
All nodes $n_i$ of a subset are used for moment-matching (e.g. if the node structure is 100-5-2 and $p^j = 0.1$ we have 10, 50, and 100 nodes available for regime $j$).

- The simulated returns $\hat{R}_n$ are randomly distributed within each of the three subsets and the subsets are randomly distributed across all nodes in that period.
- A tag is assigned to each return to identify the associated regime for later use.
- A pseudo-code included in the Appendix describes the procedure.
- Short-term (daily or weekly) volatility is highly persistent.
- After extreme events high levels of volatility are more likely than normal levels.
- However, estimating regime transition probabilities for annual or even longer intervals is not a trivial issue.
- Therefore we rather assume that the probability for ending up in a particular regime is independent of the previous regime.
- If reliable regime transition probabilities were available these could be included in applications of the model.
Point to Remember

When there is trouble in the stock market, the positive correlation between stocks and bond fails and they become negatively correlated.

When the mean of the stock market is negative, bonds are most attractive as is cash.
Between 1982 and 1999 the return of equities over bonds was more than 10% per year in EU countries

During 2000 to 2002 bonds greatly outperformed equities
Chart 2: The correlation between US equity and government bond returns

The question is whether we are moving back to a period where the two asset classes move against one another, or whether this will just prove to be a temporary phenomenon.
Statistical Properties of Asset Returns.

<table>
<thead>
<tr>
<th></th>
<th>Stocks Eur</th>
<th>Stocks Eur</th>
<th>Stocks US</th>
<th>Stocks US</th>
<th>Bonds Eur</th>
<th>Bonds US</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>monthly returns</strong></td>
<td>1/70-9/00</td>
<td>1/86-9/00</td>
<td>1/70-9/00</td>
<td>1/86-9/00</td>
<td>1/86-9/00</td>
<td>1/86-9/00</td>
</tr>
<tr>
<td>mean (% p.a.)</td>
<td>10.6</td>
<td>13.3</td>
<td>10.7</td>
<td>14.8</td>
<td>6.5</td>
<td>7.2</td>
</tr>
<tr>
<td>std.dev (% p.a.)</td>
<td>16.1</td>
<td>17.4</td>
<td>19.0</td>
<td>20.2</td>
<td>3.7</td>
<td>11.3</td>
</tr>
<tr>
<td>skewness</td>
<td>-0.90</td>
<td>-1.43</td>
<td>-0.72</td>
<td>-1.04</td>
<td>-0.50</td>
<td>0.52</td>
</tr>
<tr>
<td>kurtosis</td>
<td>7.05</td>
<td>8.43</td>
<td>5.79</td>
<td>7.09</td>
<td>3.25</td>
<td>3.30</td>
</tr>
<tr>
<td>Jarque-Bera test</td>
<td>302.6</td>
<td>277.3</td>
<td>151.9</td>
<td>155.6</td>
<td>7.7</td>
<td>8.5</td>
</tr>
</tbody>
</table>

|                |           |           |           |           |           |           |
| **annual returns** |           |           |           |           |           |           |
| mean (%)        | 11.1      | 13.3      | 11.0      | 15.2      | 6.5       | 6.9      |
| std.dev (%)     | 17.2      | 16.2      | 20.1      | 18.4      | 4.8       | 12.1     |
| skewness        | -0.53     | -0.10     | -0.23     | -0.28     | -0.20     | -0.42    |
We calculate optimal portfolios for seven cases.

- Cases with and without mixing of correlations and consider normal, t- and historical distributions.
- Cases NM, HM and TM use mixing correlations.
- Case NM assumes normal distributions for all assets.
- Case HM uses the historical distributions of each asset.
- Case TM assumes t-distributions with five degrees of freedom for stock returns, whereas bond returns are assumed to have normal distributions.
- Cases NA, HA and TA are based on the same distribution assumptions with no mixing of correlations matrices. Instead the correlations and standard deviations used in these cases correspond to an 'average' period where 10%, 20% and 70% weights are used to compute averages of correlations and standard deviations used in the three different regimes.

Comparisons of the average (A) cases and mixing (M) cases are mainly intended to investigate the effect of mixing correlations. Finally, in the case TMC, we maintain all assumptions of case TM but use Austria’s constraints on asset weights. Eurobonds must be at least 40% and equity at most 40%, and these constraints are binding.
A distinct pattern emerges:

- The mixing correlation cases initially assign a much lower weight to European bonds than the *average* period cases.
- Single-period, mean-variance optimization and the *average* period cases (NA, HA and TA) suggest an approximate 45-55 mix between equities and bonds.
- The mixing correlation cases (NM, HM and TM) imply a 65-35 mix. Investing in US Bonds is not optimal at stage 1 in none of the cases which seems due to the relatively high volatility of US bonds.
Optimal Initial Asset Weights at Stage 1 by Case (percentage).

<table>
<thead>
<tr>
<th>Case Description</th>
<th>Stocks Europe</th>
<th>Stock US</th>
<th>Bonds Europe</th>
<th>Bonds US</th>
</tr>
</thead>
<tbody>
<tr>
<td>single-period, mean-variance optimal weights</td>
<td>34.8</td>
<td>9.6</td>
<td>55.6</td>
<td>0.0</td>
</tr>
<tr>
<td>case NA: no mixing (average periods) normal distributions</td>
<td>27.2</td>
<td>10.5</td>
<td>62.3</td>
<td>0.0</td>
</tr>
<tr>
<td>case HA: no mixing (average periods) historical distributions</td>
<td>40.0</td>
<td>4.1</td>
<td>55.9</td>
<td>0.0</td>
</tr>
<tr>
<td>case TA: no mixing (average periods) t-distributions for stocks</td>
<td>44.2</td>
<td>1.1</td>
<td>54.7</td>
<td>0.0</td>
</tr>
<tr>
<td>case NM: mixing correlations normal distributions</td>
<td>47.0</td>
<td>27.6</td>
<td>25.4</td>
<td>0.0</td>
</tr>
<tr>
<td>case HM: mixing correlations</td>
<td>37.9</td>
<td>25.2</td>
<td>36.8</td>
<td>0.0</td>
</tr>
</tbody>
</table>
Expected Terminal Wealth, Expected Reserves and Probabilities of Shortfalls, Target Wealth $W_T = 206.1$

<table>
<thead>
<tr>
<th></th>
<th>Stock Europe</th>
<th>Stock US</th>
<th>Bonds Europe</th>
<th>Bonds US</th>
<th>Expected Terminal Wealth</th>
<th>Expected Reserves, Stage 6</th>
<th>Probability of Target Shortfall</th>
</tr>
</thead>
<tbody>
<tr>
<td>NA</td>
<td>34.3</td>
<td>49.6</td>
<td>11.7</td>
<td>4.4</td>
<td>328.9</td>
<td>202.8</td>
<td>11.2</td>
</tr>
<tr>
<td>HA</td>
<td>33.5</td>
<td>48.1</td>
<td>13.6</td>
<td>4.8</td>
<td>328.9</td>
<td>205.2</td>
<td>13.7</td>
</tr>
<tr>
<td>TA</td>
<td>35.5</td>
<td>50.2</td>
<td>11.4</td>
<td>2.9</td>
<td>327.9</td>
<td>202.2</td>
<td>10.9</td>
</tr>
<tr>
<td>NM</td>
<td>38.0</td>
<td>49.7</td>
<td>8.3</td>
<td>4.0</td>
<td>349.8</td>
<td>240.1</td>
<td>9.3</td>
</tr>
<tr>
<td>HM</td>
<td>39.3</td>
<td>46.9</td>
<td>10.1</td>
<td>3.7</td>
<td>349.1</td>
<td>235.2</td>
<td>10.0</td>
</tr>
<tr>
<td>TM</td>
<td>38.1</td>
<td>51.5</td>
<td>7.4</td>
<td>2.9</td>
<td>342.8</td>
<td>226.6</td>
<td>8.3</td>
</tr>
<tr>
<td>TMC</td>
<td>20.4</td>
<td>20.8</td>
<td>46.3</td>
<td>12.4</td>
<td>253.1</td>
<td>86.9</td>
<td>16.1</td>
</tr>
</tbody>
</table>

If the level of portfolio wealth exceeds the target, the surplus is allocated to a reserve account and a portion used to increase [10% usually] wealth targets.
optimal allocations, expected wealth and shortfall probabilities are mainly affected by considering mixing correlations while the type of distribution chosen has a smaller impact. This distinction is mainly due to the higher proportion allocated to equities if different market conditions are taken into account by mixing correlations.
Effect of the Risk Premium: Differing Future Equity Mean Returns

- mean of US stocks 5-15%.
- mean of European stocks constrained to be the ratio of US/European
- mean bond returns same
- case NM (normal distribution and mixing correlations).
- As expected, [Chopra and Ziemba (1993)], the results are very sensitive to the choice of the mean return.
  - If the mean return for US stocks is assumed to equal the long run mean of 12% as estimated by Dimson et al. (2002), the model yields an optimal weight for equities of 100%.
  - a mean return for US stocks of 9% implies less than 30% optimal weight for equities
Optimal Asset Weights at Stage 1 for Varying Levels of US Equity Means

Observe extreme sensitivity to mean estimates
The Effects of State Dependent Correlations

Optimal Weights Conditional on Quintiles of Portfolio Wealth at Stage 2 and 5

A. Stage 2

Average Wealth in Quintile

- U.S. Bonds
- European Bonds
- U.S. Equities
- European Equities

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• Average allocation at stage 5 is essentially independent of the wealth level achieved (the target wealth at stage 5 is 154.3)

• The distribution at stage 2 depends on the wealth level in a specific way.
  
  • Slightly below target (103.4) a very cautious strategy is chosen. Bonds have a weight highest weight of almost 50%. The model implies that the risk of even stronger underachievement of the target is to be minimized and it relies on the low but more certain expected returns of bonds to move back to the target level.
  
  • Far below the target (97.1) a more risky strategy is chosen. 70% equities and a high share (10.9%) of relatively risky US bonds. With such strong underachievement there is no room for a cautious strategy to attain the target level again.
  
  • Close to target (107.9) the highest proportion is invested into US assets with 49.6% invested in equities and 22.8% in bonds. The US assets are more risky than the corresponding European assets which is acceptable because portfolio wealth is very close to the target and risk does not play a big role.
  
  • Above target most of the portfolio is switched to European assets which are safer than US assets. This decision may be interpreted as an attempt to preserve the high levels of attained wealth.
decision rules implied by the optimal solution can test the model using the following rebalancing strategy.

Consider the ten year period from January 1992 to January 2002.

- first month assume that wealth is allocated according to the optimal solution for stage 1
- in subsequent months the portfolio is rebalanced
  - identify the current volatility regime (extreme, highly volatile, or normal) based on the observed US stock return volatility.
  - search the scenario tree to find a node that corresponds to the current volatility regime and has the same or a similar level of wealth.
  - The optimal weights from that node determine the rebalancing decision.
  - For the no-mixing cases NA, TA and HA the information about the current volatility regime cannot be used to identify optimal weights. In those cases we use the weights from a node with a level of wealth as close as possible to the current level of wealth.
Cumulative Monthly Returns for Different Strategies.
Conclusions and final remarks

- Stochastic Programming ALM models are useful tools to evaluate pension fund asset allocation decisions.
- Multiple period scenarios/fat tails/uncertain means.
- Ability to make decision recommendations taking into account goals and constraints of the pension fund.
- Provides useful insight to pension fund allocation committee.
- Ability to see in advance the likely results of particular policy changes and asset return realizations.
- Gives more confidence to policy changes
The following quote by Konrad Kontriner (Member of the Board) and Wolfgang Herold (Senior Risk Strategist) of Innovest emphasizes the practical importance of InnoALM:

“The InnoALM model has been in use by Innovest, an Austrian Siemens subsidiary, since its first draft versions in 2000. Meanwhile it has become the only consistently implemented and fully integrated proprietary tool for assessing pension allocation issues within Siemens AG worldwide. Apart from this, consulting projects for various European corporations and pensions funds outside of Siemens have been performed on the basis of the concepts of InnoALM.

The key elements that make InnoALM superior to other consulting models are the flexibility to adopt individual constraints and target functions in combination with the broad and deep array of results, which allows to investigate individual, path dependent behavior of assets and liabilities as well as scenario based and Monte-Carlo like risk assessment of both sides.

In light of recent changes in Austrian pension regulation the latter even gained additional importance, as the rather rigid asset based limits were relaxed for institutions that could prove sufficient risk management expertise for both assets and liabilities of the plan. Thus, the implementation of a scenario based asset allocation model will lead to more flexible allocation restraints that will allow for more risk tolerance and will ultimately result in better long term investment performance.

Furthermore, some results of the model have been used by the Austrian regulatory authorities to assess the potential risk stemming from less constraint pension plans.”
Point 1 - Means are by far the most important part of the distribution of returns – especially the direction.

→ You must estimate future means well or you can travel very fast in the wrong direction and that usually leads to losses or underperformance. The effect is risk aversion dependent. The less risk averse you are the bigger is the effect of mean estimation errors on performance.

Point 2 - Mean variance models are useful as a basic guideline when you are in an assets only situation. Professionals adjust means using mean-reversion, James-Stein or truncated estimators and constrain output weights.

→ Do not change asset positions unless the advantage of the change is significant. Do not use mean variance analysis with liabilities and other major market imperfections except as a first test analysis.

Point 3 - Trouble arises when one overbets and a bad scenario occurs

→ You must not overbet when there is any possibility of a bad scenario occurring unless the bet is protected by some type of hedge or stop-loss. Mental stop-losses are often better than market ordered stops.
The top ten points to remember about the stochastic programming approach to asset liability and wealth management.

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- You must estimate future means well or you can travel very fast in the wrong direction and that usually leads to losses or underperformance. The effect is risk aversion dependent. The less risk averse you are the bigger is the effect of mean estimation errors on performance.

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Mental stop losses are often better than market ordered stops
**Point 4** - Trouble is exacerbated when the expected diversification does not hold in the scenario that occurs

→ You must use scenario dependent correlation matrices because simulations around historical correlation matrices are inadequate for extreme scenarios.

**Point 5** - When there is a large decline in the stock market, the positive correlation between stocks and bond fails and they become negatively correlated

→ When the mean of the stock market is negative, bonds are usually more attractive as is cash. So the correlation is negative.

**Point 6** - Stochastic programming scenario based models are useful when one wants to look at *aggregate* overall decisions with liabilities, liquidity, taxes, policy, legal and other constraints and have targets and goals you want to achieve.

→ It pays to make a complex stochastic programming model when *a lot is at stake* and the essential problem has many complications.
Point 7 - Other approaches, continuous time finance, decision rule based SP, control theory, etc are useful for problem insights and theoretical results. But in actual use, they may lead to disaster unless modified. The Black-Scholes theory says you can hedge perfectly with log normal assets and this can lead to overbetting.

But fat tails and jumps arise frequently and can occur without warning. The S&P opened limit down –60 points or 6% when trading resumed after Sept 11, 2001 and it fell 14% that week.

→ Be careful of the assumptions, including implicit ones, of theoretical models. Use the results with caution no matter how complex and elegant the math or how smart or famous the author. Remember you have to be very smart to lose millions and even smarter to lose billions.
Point 8 - Do not be concerned with getting all the scenarios exactly right when using stochastic programming models.

You cannot do this and it does not matter that much anyway.

Rather worry that you have the problems’ periods laid out reasonably and the scenarios basically cover the means, the tails and the chance of what could happen. If the current situation has never occurred before, use one that is similar to add scenarios.

For a crisis in Brazil, use Russian crisis data, for example. The results of the SP will give you good advice when times are normal and keep you out of severe trouble when times are bad.

Those using SP models may lose 5-10-15% but they will not lose 50-70-95% like some investors and hedge funds.

→ If the scenarios are more or less accurate and the problem elements reasonably modeled, the SP will give good advice.

You may slightly outperform in normal markets but you will greatly outperform in bad markets when other approaches may blow up.

Current markets are also greatly affected by political concerns and statements by Central Bank officials and country lenders.
Point 9 - SP models for ALM were very expensive in the 1980s and early 1990s but are not very expensive now. Vancouver analysts using a large linear programming model to plan lumber operations at MacMillan Bloedel used to fly to San Francisco to use a large computer which would run all day to run the model once. Now models of this complexity would take only seconds on an inexpensive desk top computer.

→ Advances in computing power and modeling expertise have made SP modeling not very expensive.

Such models are still complex and require approximately six months to develop and test, costing a couple hundred thousand dollars. A small team can now make a model for a complex organization quite quickly at fairly low cost compared to what is at stake.
Point 10 - Eventually as there are more disasters and more successful SP models are built and used, they will become popular.

→ The ultimate goal is to have them in regulations like VaR. While VaR does more good than harm, it's safety is questionable in many applications. C-VaR is an improvement but for most people and organizations the non-attainment of goals is more than proportional, that is, convex in the non-attainment.
Comments on Scenarios

1. Idea to cover the range of possible futures
2. N periods: earlier periods are easier to predict; further out periods are much harder to predict but then we have the past which can be helpful
3. Mean most important 20-2-1 and risk aversion dependent
   Low Arrow-Pratt risk aversion implies 100-3-1 for log with $R_A=1/w$
   see Chopra and Ziemba (1993) reprinted in Ziemba, *Great Investment Strategies*
4. You cannot get it perfect but it does not matter, an important thing is to not assign zero probability to extremely rare possibilities.
5. Even if the scenarios are not “good” they still improve decision making performance
6. We have
   a. Past data and econometrics
   b. Expert opinion
   c. Models, here are lots of models for predictions in the *Crash* book
   d. This is a whole subfield of stochastic programming
   e. Now politics in the current era
      this is very useful in short term derivative trading and market timing strategies
      my two greatest trading days ever were the nights of the US Trump and French
      Macron elections, see Chapter 8 in *Crashes big and small and what to do about them*, Ziemba, Lleo and Zhitlukhin
   f. Scenario dependent correlations are very important as we saw in the Vienna
      InnoALM model
Books relevant to ALM management