The P2P pandemic swap: decentralized pandemic-linked securities

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Pandemic risk management

Insurability of pandemic risks

- Pandemic risk is **systematic**
  - A pandemic crosses national borders.
  - Pandemic losses in different countries are strongly positive dependent.
  - Diversification of pandemic risks is difficult.

- **Heterogeneous** risks:
  - Each country has its own severity distribution.
  - When and how much extra capital is needed depends on the country.

- **Size** of the pandemic losses
  - exceeds the capacity of the insurance market;
  - Pandemic risks **cannot** be fully covered by traditional insurance.
  - German Insurance Association (2020) and Richter & Wilson (2020).
1 – Pandemic risk management

Solutions

- **Solution 1:** Peer-to-peer network of countries
  - Countries pool (part of) their pandemic losses together
  - and each country pays a predetermined share of the losses.
  - P2P networks can cope with heterogeneous risks.
  - Abdikerimova & Feng (2022) and Denuit, Dhaene & Robert (2022).

- **Solution 2:** Insurance-linked securities
  - Similar to CAT bonds, one can transfer the pandemic losses to the financial market using insurance-linked securities.
  - The capacity of the financial market can be used for absorbing the pandemic losses.
  - Benefits to investors
    - Investors receive a periodical premium.
    - A pandemic bond may be used to diversify a portfolio.
Pandemic Financing Facility Fund\(^2\):
- Goal: Transfer funds to poor countries in case they are hit by a major pandemic.
- A pandemic bond was introduced to provide the insurance coverage.

Pandemic bond:
- Investors fund the World Bank by paying a principal at initiation.
- Donor countries (Australia, Germany and Japan) provide a series of coupon payments to compensate the investors.
- In case of a pandemic, (part of) the principal is used to fund countries in need to respond to the pandemic.

\(^2\)More information can be found [here](#)
Drawbacks of the World Banks’ pandemic bond

- **Slow triggering mechanism:**
  - The bond only paid out in 3 cases out of 60 pandemics.
  - More than 100 million USD was paid out to investors via the coupons.

- **Donor fatigue:**
  - The 3 donor countries only contribute and have no future benefits.
  - The receiving countries only receive in case of a pandemic.

- **Triggers are not country specific:**
  - The bond cannot differentiate between countries.
We introduce the class of **P2P Pandemic-linked securities.**

- Transfer part of the pandemic risk to the financial market:
  - similar to CAT bonds, longevity bonds, etc.
- Use a peer-to-peer network between countries.
  - mutual support between countries.
3 – The P2P pandemic swap

Structure

- **Participants:**
  - **investors:** provide insurance against pandemic risk in return for a periodic premium.
  - **Pool of countries:** collectively pay the insurance premium and support each other in case of a pandemic.

- **Payments:**
  - **Fixed periodical premiums:** Transferring the losses to the investors will require paying a fixed premium.
  - **Random losses:** Whenever a pandemic loss occurs, part of these random losses will be covered by the investors.
The countries are organised in a P2P network
- $s_j$: benefit of country $j$.
- $\alpha_{ij}$: proportion country $i$ pays to country $j$.

$\alpha_{ij} \times s_j = $ Loss payment of country $i$ to country $j$.

Pandemic swap:
- Insurance for the losses which are not covered by the pool.

$\alpha_{0j} \times s_j = $ Amount the investors pay to country $j$.

Max amount covered by the bond:

$$ F = \sum_{j=1}^{n} \alpha_{0j}s_j. $$
3 – The P2P pandemic swap

The investors

- **Premium Income:**
  - Payment dates:
    \[ t_1 < \ldots < t_N = T. \]
  - Equidistant time grid: \( \Delta_t = t_2 - t_1. \)
  - The pool of countries collectively fund the premiums:
    \[
cF \Delta_t = \text{Premium paid at each payment date}
\]

- **Benefit payments:**
  - Premium payments stop when the *first payment* is triggered.
  - The investors pay part of the benefit amount.
  - The maximal amount paid by the investors is \( F. \)
Conditions for the payments

- **Conservation of zero balance for risk sharing**

\[
\sum_{i=0, i\neq j}^{n} \alpha_{ij} = 1, \quad \text{for } j = 1, 2, \ldots, n. \tag{1}
\]

- The contributions of the investors and countries are sufficient to cover the payment \( s_j \) to country \( j \).

- **Collective payment of premiums**

\[
\sum_{i=1}^{n} \alpha_{i0} = 1. \tag{2}
\]

- The aggregate contributions of the countries are sufficient to cover the premium payment \( Fc\Delta t \).
Conditions for the payments

- **Principle of indemnity**

  \[ 0 \leq \alpha_{ij} \leq 1, \quad i, j \geq 0. \]  

  ▶ The amount country \( i \) pays to country \( j \) in case of a pandemic event, should never exceed the insured amount \( s_j \).

  ▶ No one should make a profit, i.e. \( \alpha_{ij} \geq 0 \).

- **Maximum principal loss.**

  \[ \sum_{j=1}^{n} s_j \alpha_{0j} = F. \]

  ▶ In the most extreme event where all countries will be triggered, the full amount \( F \) will be used.
Define the process \( \{ J_i(t) \mid t \in [0,T] \} \) as follows:

\[
J_i(t) = \begin{cases} 
0 & \text{if a payment for country } i \text{ is not triggered before } t, \\
1 & \text{if a payment for country } i \text{ is triggered before } t.
\end{cases}
\]

Define for each payment date \( t_j \):

\[
I_i(t_j) = J_i(t_j) - J_i(t_{j-1}).
\]

- \( I_i(t_j) = 1 \): a payment is triggered for country \( i \) in the interval \([t_{j-1}, t_j]\),
- and country \( i \) will receive the benefit amount \( s_i \) at time \( t_j \).
Define:

\[ I_0(t) = \prod_{i=1}^{n} (1 - J_i(t)). \]

- If \( I_0(t_j) = 1 \):
  - no payments are triggered before \( t_j \).
  - Investors receive the premium payment \( F c \Delta_t \) at the payment date \( t_j \).

- If \( I_0(t_j) = 0 \):
  - At least one country received a benefit payment in the interval \( [0, t_j] \).
  - There are no premium payments during the remaining lifetime of the swap.
Example 1: The WHO pandemic bond

- No payments between countries:
  \[ \alpha_{ij} = 0 \text{ for } i, j = 1, 2, \ldots, n. \]

- The first 3 countries are the donor countries:
  \[ \alpha_{i0} = 0, \text{ for } i = 4, 5, \ldots, n. \]

  Donor countries will not receive any benefits: \( s_1 = s_2 = s_3 = 0 \).

- The recipient countries receive the benefit payments from the investors:
  \[ \alpha_{0i} = 1, \text{ for } i = 4, 5, \ldots, n, \]
  and
  \[ \alpha_{01} = \alpha_{02} = \alpha_{03} = 0. \]
Example 2: P2P bond with 2 countries

Consider a pandemic swap with 2 countries.

- Payments between countries: $\alpha_{12}$ and $\alpha_{21}$.
- Payments from the investors to the countries: $\alpha_{01}$ and $\alpha_{02}$.
- Premium payments: $\alpha_{10}$ and $\alpha_{20}$.

Conditions on the premium payments:

$$\alpha_{10} + \alpha_{20} = 1.$$ 

- The countries can decide how to share the premium payments.
- The premium payments can be determined separately from the loss payments.
The payments from the countries should satisfy the conditions (1), (3) and (4):

**Set of possible solutions:**

\[\alpha_{21} = \frac{s_1 + s_2 - F - s_2 \alpha_{12}}{s_1},\]
\[\alpha_{01} = 1 - \alpha_{21},\]
\[\alpha_{02} = 1 - \alpha_{12},\]

In order to have \(0 \leq \alpha_{ij} \leq 1:\)

\[\max \left(\frac{s_2 - F}{s_2}, 0\right) \leq \alpha_{12} \leq \min \left(\frac{s_1 + s_2 - F}{s_2}, 1\right).\]
Example 2: P2P bond with 2 countries (cont’d)

![Graph showing benefit transfers $\alpha_{01}$ (black), $\alpha_{02}$ (green), and $\alpha_{21}$ (blue) in function of $\alpha_{21}$.]

$s_1 = 100$, $s_2 = 200$ and $F = 150$. 

**Figure.** The benefit transfers $\alpha_{01}$ (black), $\alpha_{02}$ (green) and $\alpha_{21}$ (blue) in function of the transfer $\alpha_{21}$. 

$s_1 = 100$, $s_2 = 200$ and $F = 150$. 
Example 2: P2P bond with 2 countries (cont’d)

- If $\alpha_{12}$ increases:
  - Country 1 pays a larger share of the benefit of country 2 in case of a pandemic event.
  - The benefit amount $s_2$ is fixed, therefore the share $\alpha_{02}$ the investors pay decreases.

- If $\alpha_{02}$ decreases:
  - less of the total available funds $F$ of the investors is attributed to country 2,
  - hence more will be allocated to country 1,
  - Then $\alpha_{01}$ increases.

- If $\alpha_{01}$ increases:
  - the investors pay a larger share of the loss of country 1,
  - so the part country 2 pays decreases,
  - hence $\alpha_{21}$ decreases.
The cash flow of country $i$ at time $t_j$:

$$R_i(t_j) = s_i I_i(t_j) - \alpha_{i0}Fc \Delta t I_0(t_j) - \sum_{k=1, k \neq i}^{n} \alpha_{ik}s_k I_k(t_j).$$

- The benefit payment in case of a triggering pandemic event.
- The premium payment in case no payment was yet triggered.
- P2P payments to other countries.

The time-0 cash flow for country $i$:

$$R_i = \sum_{j=1}^{N} e^{-rt_j} R_i(t_j),$$

where $r$ is the risk-free rate which is assumed to constant.
The expected return for the countries and the investors

- **Rewrite** $R_i$:

\[
R_i = s_i I_i - \alpha_{i0} Fc \Delta t I_0 - \sum_{k=1, k \neq i}^{n} \alpha_{ik} s_k I_k,
\]

where $I_i = \sum_{j=1}^{N} e^{-r_j t_j} I_i(t_j)$.

- **Notation**:

\[
\mathbb{E}[I_i] = q_i, \text{ for } i = 1, \ldots, n.
\]

\[
\mathbb{E}[I_0] = p_0.
\]

- **Expected return for country $i$**:

\[
\mathbb{E}[R_i] = s_i q_i - \alpha_{i0} (Fc \Delta t) p_0 - \sum_{k=1, k \neq i}^{n} \alpha_{ik} s_k q_k.
\]
Fairness of a P2P pandemic swap:

- The P2P pandemic swap is **fair** if the expected return for each country is zero:

\[ \mathbb{E}[R_i] = 0, \text{ for } i = 1, 2, \ldots, n. \]

Result:

- The expected time-0 return of the investors

\[ \mathbb{E}[R_0] = (Fc\Delta t)p_0 - \sum_{k=1}^{n} s_k \alpha_{0k} q_k . \]

- If the P2P bond is fair, we have that \( \mathbb{E}[R_0] = 0. \)
Allocate the available capital $F$ to the different countries.

Determine $\alpha_0, \alpha_1, \ldots, \alpha_n$ such that

$$\sum_{i=1}^{n} \alpha_0 s_i = F.$$ 

The fairness condition for the investors implies that

$$c = \frac{\sum_{k=1}^{n} s_k \alpha_0 q_k}{F p_0 \Delta t}.$$ 

Putting more weight on risky countries, i.e. countries with higher trigger probability, will increase the riskiness of the pandemic swap, which increases the premium $c$. 
Countries and investors should cover the full loss amount.

\[ \sum_{i=1, i \neq k}^{n} \alpha_{ik} = 1 - \alpha_{0k}, \text{ for } k = 1, 2, \ldots, n. \]

- \( \alpha_{ik} \) = the proportion of the loss of country \( k \) paid by country \( i \).
- \( n \) equations and \( n(n-1) \) unknowns.

**Fairness conditions:**

\[ \alpha_{i0} = \frac{s_i q_i - \sum_{k=1, k \neq i}^{n} \alpha_{ik} s_k q_k}{F c \Delta t p_0}. \]

- We have that \( \sum_{i=1}^{n} \alpha_{i0} = 1. \)
We solve the following problem

\[\begin{align*}
\text{maximize} \quad & f = \sum_{i=1}^{n} \sum_{j \neq i}^{n} v(\alpha_{ij}) \\
\text{subject to} \quad & \sum_{i=1}^{n} \sum_{j \neq i}^{n} \alpha_{ij} = 1 - \alpha_{0k}, \text{ for } k = 1, 2, \ldots, n. \\
& \sum_{k=1}^{n} \alpha_{ik}s_kq_k \leq s_iq_i, \text{ for } i = 1, 2, \ldots, n.
\end{align*}\]

We maximize the mutual support between countries:

- \( v \) is a concave and non-decreasing function.
Closed-form solution

- Assume:

\[\sum_{\substack{k=1 \atop k \neq i}}^{n} (1 - \alpha_{0k}) s_k q_k \leq (n - 1) s_i q_i, \text{ for } i = 1, 2, \ldots, n.\]

- A solution to the maximization problem:

\[\alpha_{ik} = \frac{1 - \alpha_{0k}}{n - 1}, \text{ for } i = 1, 2, \ldots, n.\]
The time that the payment for country $i$ is triggered is $\tau_i$.

Denote the intensity for country $i$ by $\lambda_i$:

$$\Pr[\tau_i > t] = e^{-\lambda_i t}.$$  

Then:

$$q_i = \frac{(1 - e^{-\lambda_i \Delta t}) e^{-(\lambda_i + r) \Delta t} (1 - e^{-(\lambda_i + r) T})}{1 - e^{-(\lambda_i + r) \Delta t}}.$$  

In order to model the premium payments, we need the dependence structure between the random variables $\tau_i$. 
An intensity model: dependence

- **Ordered probabilities:**

  \[ e^{-\lambda_1} \geq e^{-\lambda_2} \geq \ldots \geq e^{-\lambda_n}. \]

  - Country 1 is the safest country. Country \( n \) is the riskiest.

- **We assume:**

  \[ \mathbb{P}[\tau_{i+1} \leq t | \tau_i \leq t] = 1, \text{ for } i = 1, 2, \ldots, n - 1. \]

  - If a payment for country \( i \) was triggered before \( t \), all riskier countries also received their benefit payment before time \( t \).
Triggers are ordered:

- The first country to receive a benefit payment is the riskiest country, followed by the 2nd riskiest country, etc.
- See also Dhaene & Goovaerts (1997).

Premium payments:

\[ \mathbb{E} [I_0] = p_0 = \frac{e^{-(\lambda_n + r)\Delta t} \left(1 - e^{-(\lambda_n + r)T}\right)}{(1 - e^{-(\lambda_n + r)\Delta t})} . \]

- The expectation only depends on the intensity of the riskiest country.
The single-trigger case

Assume a single trigger:

\[ I_i = I, \text{ for } i = 1, 2, \ldots, n. \]

- The probability and moment of triggering a pandemic loss payment is the same for each country.
- If the P2P pandemic swap pays losses, it will pay to all countries at a single moment.
- However, the insured amounts \( s_i \) can be different.
6 – Examples
The single-trigger case

- **Coupon:**

  \[ c = \frac{q}{\Delta_t p_0} \approx \lambda. \]

  - \( \lambda \): the intensity of the single trigger.

  - The P2P pandemic swap behaves as a defaultable bond with zero recovery; see e.g. De Spiegeleer & Schoutens (2019).

- **Payments of the countries:**

  \[ \alpha_{i0} = \frac{s_i - \sum_{k=1}^{n} \alpha_{ik} s_k}{F}. \]
Consider a group of $n$ countries.

**Single-trigger mechanism:**

$$I_i = I, \text{ for } i = 1, 2, \ldots, n.$$ 

**Benefits are the same:**

$$s_i = s, \text{ for } i = 1, 2, \ldots, n.$$ 

**Coupon payments:** $\alpha_{10}$

- Each country pays the same share $\alpha_{10}$ of the total coupon payment:

$$\alpha_{10} = \frac{1}{n}$$
The Homogeneous case

- **Investors:**
  - In case a country is eligible to receive a loss payment, the investors pay the proportion $\alpha_0$:
    \[ \alpha_0 = \frac{F}{sn}. \]

- **Mutual support: $\alpha_1$**
  - Each country pays the same amount $\alpha_1 \times s$ to cover losses of other countries:
    \[ \alpha_1 = \frac{1}{n-1} - \frac{F}{sn(n-1)}. \]

- In the homogeneous case, the payments $\alpha$ are determined using the feasibility constraints.
Consider two countries and a fair P2P pandemic swap.

\[
\alpha_{12} = 1 - \frac{F}{s_2} \left( \frac{c\Delta t p_0 - q_1}{q_2 - q_1} \right)
\]

\[
\alpha_{21} = 1 - \frac{F}{s_1} \left( \frac{q_2 - c\Delta t p_0}{q_2 - q_1} \right)
\]

\[
\alpha_{10} = \frac{1}{F c \Delta t p_0} \left( s_1 q_1 - s_2 q_2 + F q_2 \frac{c\Delta t p_0 - q_1}{q_2 - q_1} \right)
\]

\[
\alpha_{20} = 1 - \frac{1}{F c \Delta t p_0} \left( s_1 q_1 - s_2 q_2 + F q_2 \frac{c\Delta t p_0 - q_1}{q_2 - q_1} \right)
\]

\[
\alpha_{01} = \frac{F}{s_1} \left( \frac{q_2 - c\Delta t p_0}{q_2 - q_1} \right)
\]

\[
\alpha_{02} = \frac{F}{s_2} \left( \frac{c\Delta t p_0 - q_1}{q_2 - q_1} \right).
\]
Figure. Solid lines: payments of the investors to country 1 (blue) and country 2 (red). Dashed lines are the payments between countries.
Figure. The proportion of the premium payment paid by country 1 (blue) and country 2 (red).
Figure. The degree of mutual support between countries.
Thank you for your attention!

And now, the end is near, and so I face the final curtain ...


German Insurance Association (GDV). 2020. Green paper—Supporting the economy to better cope with the consequences of future pandemic events.


