

What is Quantum Computing ?

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Alonso Peña



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Quantitative Analyst at European Investment Bank

Luxembourg, Luxembourg, Luxembourg · [Contact info](#)

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Quantitative Analyst

European Investment Bank (EIB)
Aug 2019 - Present · 4 yrs 4 mos
Luxembourg

Officer Credit Risk Management



SDA Professor (Banking and Insurance Department)

SDA Bocconi School of Management
May 2011 - Jul 2019 · 8 yrs 3 mos
Milan Area, Italy



Quantitative Analyst (Structured Products)

Thomson Reuters
2007 - 2011 · 4 yrs



Quantitative Analyst (Model Validation)

Unicredit
2003 - 2007 · 4 yrs



Wellcome Trust Training Fellow (Mathematical Biology), Senior Research Associate, Research Associate

University of Cambridge
1997 - 2003 · 6 yrs
University of Cambridge

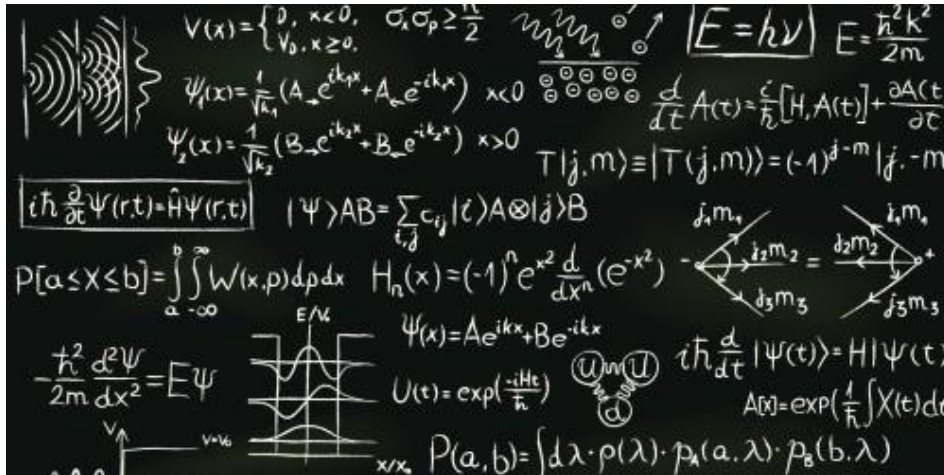
European Investment Bank (EIB)



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Quantum Computing: Introduction

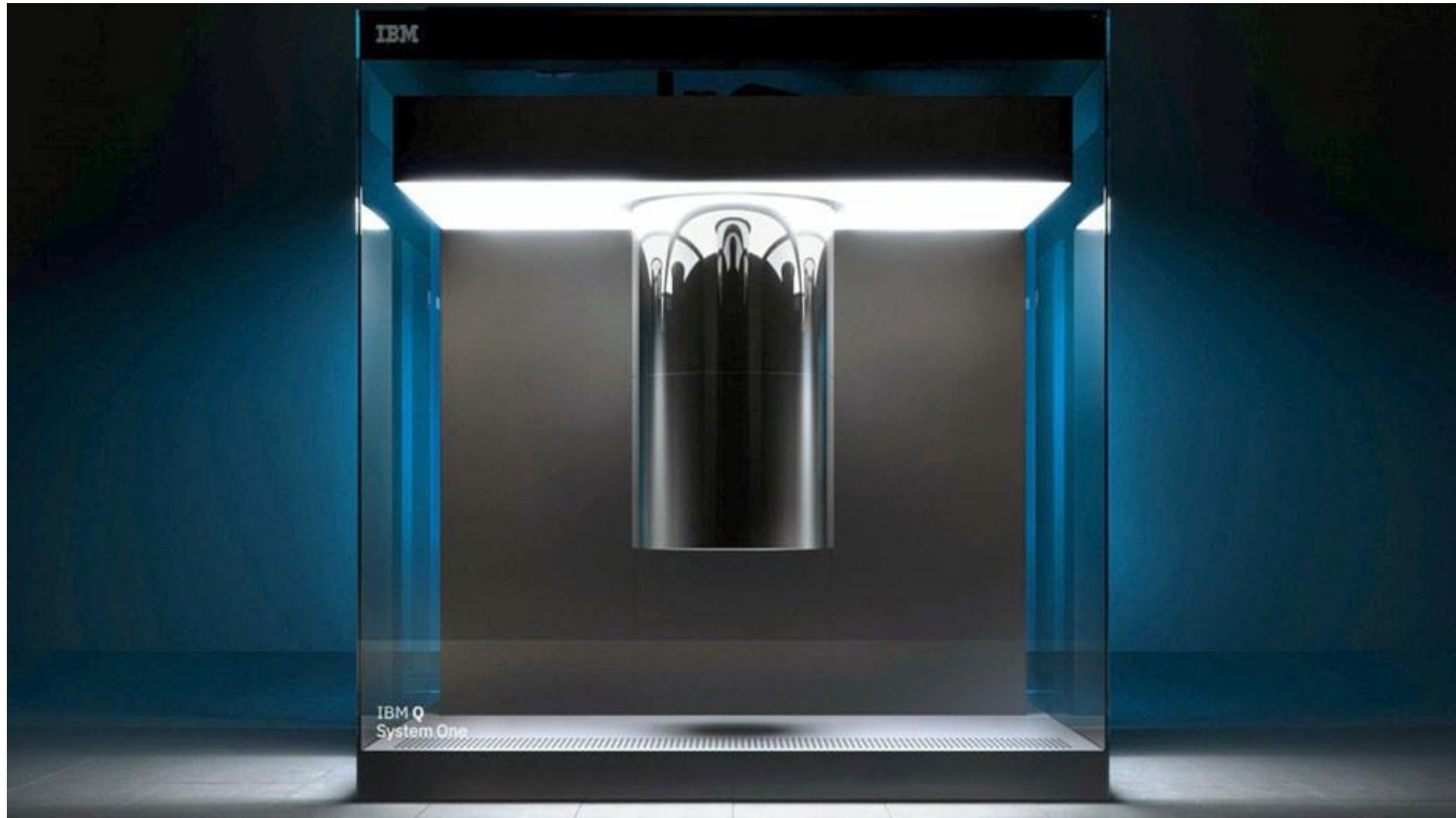
Quantum Physics



Computing



What if ?



IBM System One: the first commercial quantum computer, 2019



President Trump with the signed *National Quantum Initiative Act* (2019)



NATIONAL QUANTUM INITIATIVE

THE FEDERAL SOURCE AND GATEWAY TO QUANTUM R&D ACROSS THE U.S. GOVERNMENT

Welcome to *quantum.gov*, the home of the National Quantum Initiative and its ongoing activities to explore and promote Quantum Information Science (QIS). The [National Quantum Initiative Act](#) provides for the continued leadership of the United States in QIS and its technology applications. It calls for a coordinated Federal program to accelerate quantum research and development for the economic and national security of the United States. The United States strategy for QIS R&D and related activities is described in the [National Strategic Overview for QIS](#) and [supplementary documents](#).

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- [The Role of International Talent in Quantum Information Science](#), October 5, 2021
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- [Quantum Frontiers Report](#), October 7, 2020
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www.quantum.gov



AGENCIES



Rectangular Snip



From Academia to Industry

1981

May 6, 1981. Boston, MA, USA

Professor Richard Feynman
from Caltech, is about to give a
keynote speech in a conference
at MIT

The Nobel laureate will present
an idea for a revolutionary new
type computer, a **quantum
computer**





Feynman's idea was to move away from the **binary representation** of information.

He argued that the basic building block of information ought to be the individual **subatomic particles**.

In fact, to use the unit of information used by nature.

2019

January 8, 2019. Yorktown Heights, New York, USA

The company International Business Machines (IBM) presents the first **commercial quantum computer** in the world.

www.quantum-computing.ibm.com





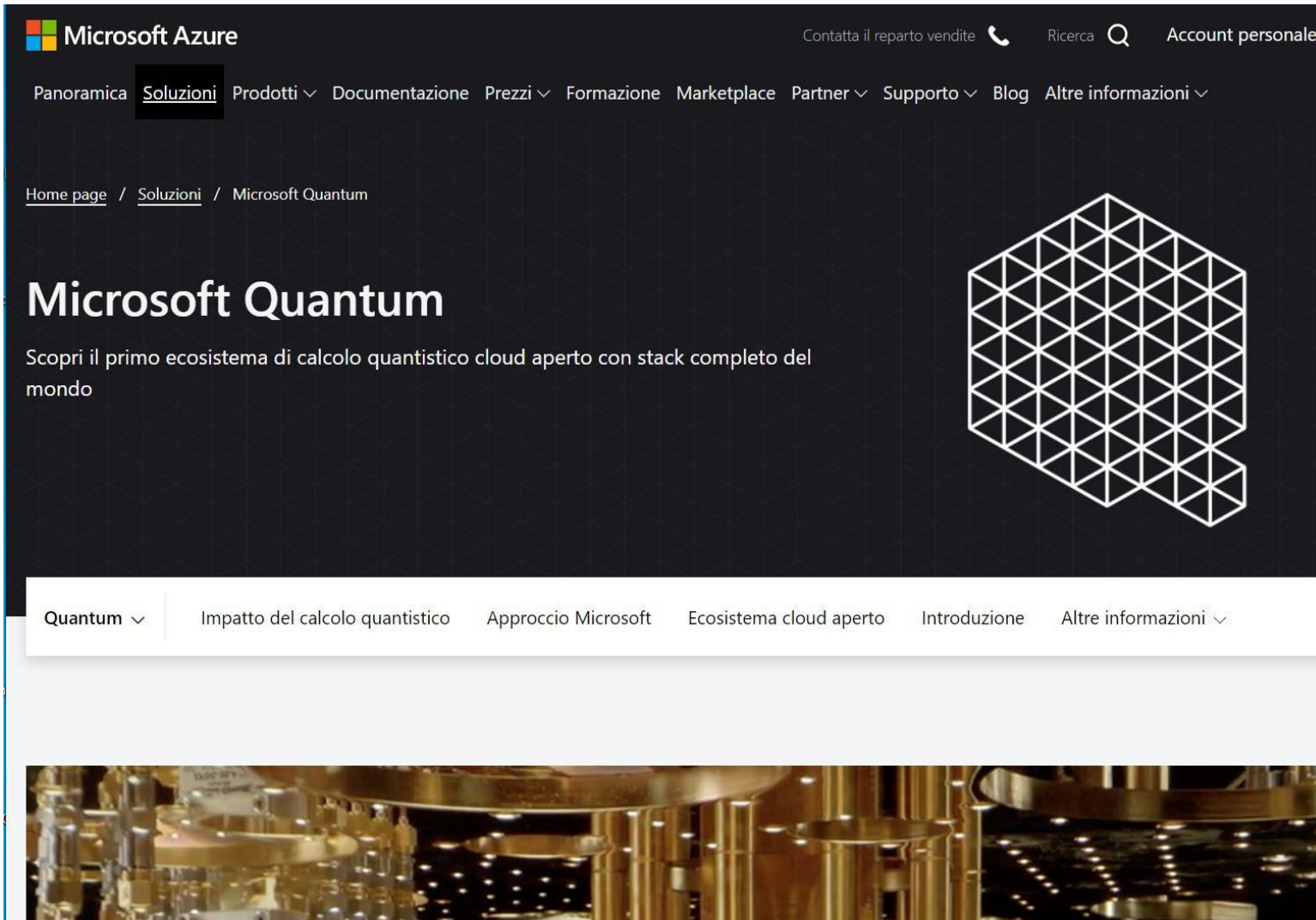
Scientists at Google Research in California managed to have their quantum system perform a mathematical calculation in **200 seconds that today's most powerful supercomputers would need more than 10,000 years to complete.**

Quantum supremacy using a programmable superconducting processor

Frank Arute et al.

Nature volume 574, pages505–510 (2019)





Microsoft Corporation created the **Microsoft Quantum Network** and Q# programming language for quantum computers.

<https://www.microsoft.com/en-us/quantum>

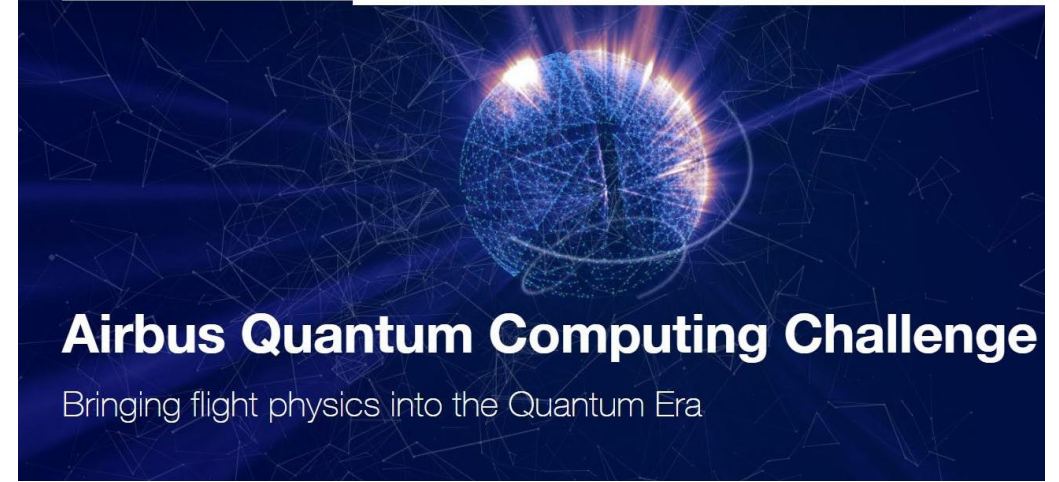
Quantum Computing: Industrial Applications



Recalculating the future of drug development with quantum computing

October 23, 2020 | Interview

pharmaceutical



Airbus Quantum Computing Challenge

Bringing flight physics into the Quantum Era

aerospace engineering



automotive (electric)



cybersecurity

How quantum computers could help design airplanes

The Boeing Company is looking toward a future where their engineers use quantum computers to help design airplanes.

IBM and Boeing chart a streamlined quantum approach to one of the biggest challenges in aerospace engineering

Watch the video



VIDEO: Boeing seeks new ways to engineer strong, lightweight materials

<https://youtu.be/BVG97KD9qVg>



Why quantum mechanics will be key to digital security

Banking



Quantum computing

Researchers have known about the theoretical potential of quantum computing for decades, but it is only in recent years that quantum computers have been developed with sufficient power to start exploiting the technology.

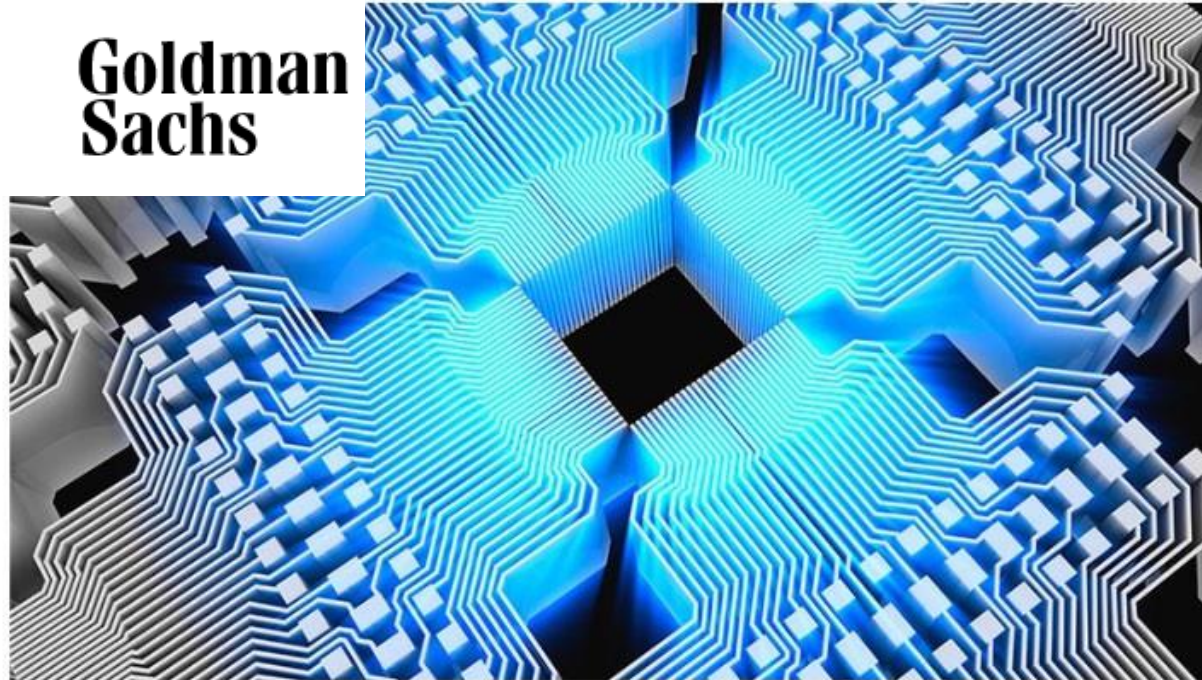
In essence, quantum computing is a powerful new technology that will allow us to solve certain problems that are more complicated than classical computers currently allow. For example, it could potentially complete complex calculations in seconds that could take years to finish on a classical computer.

This can be a challenging topic to get to grips with from a beginner's perspective, so in the boxes below, we have given an easy-to-understand introduction to the technology. They cover an overview of the fundamental building blocks of quantum computing, and provide answers to some of the most frequently answered questions about the technology.

Digital security

Banking

Engineering Quantum Algorithms

Goldman
Sachs

Quantum

Researchers have known
is only in recent years that
exploiting the technology.

In essence, quantum computing
problems that are more complex
could potentially complete
classical computer.

This can be a challenging task.
below, we have given an
overview of the fundamental
some of the most frequent

Goldman Sachs introduces quantum algorithms developed by its Research and Development Engineering team that could allow the firm to price financial instruments at quantum speeds.

Quantum algorithms could do complex financial calculations with blazing speed. Finance was one of the first domains to embrace Big Data, and the drive to innovate continues. Much of the science behind the pricing of financial assets involves simulating large numbers of different statistical

Banking



Engineering Quantum Algorithms

Goldman Sachs



HSBC Working with IBM to Accelerate Quantum Computing Readiness

Bank envisions application of quantum capabilities for priorities such as pricing and portfolio optimisation, sustainability, risk and fraud

Expands internal talent with quantum specialists

Mar 29, 2022



Quantum

Researchers have known for decades that quantum computing is only in recent years that we are beginning to exploit the technology.

In essence, quantum computing can solve problems that are more complex than a classical computer could potentially complete.

This can be a challenging task. Below, we have given an overview of the fundamental concepts and some of the most frequent applications.

Goldman Sachs introduced a Quantum and Development Engineering team to focus on financial instruments and risk management.

Quantum algorithms could be one of the first domains to exploit the science behind the pricing of derivatives.

Banking



Engineering Quantum Algorithms

Goldman Sachs



HSBC Wo Quantum

*Bank envisions app
optimisation, sustai*

Expands internal ta

Mar 29, 2022

J.P.Morgan | CHASE

TECHNOLOGY

Global Technology Applied Research

Conducting applied research focused on frontier technologies to transform scientific findings into business value.

Quantum

Researchers have known
is only in recent years the
exploiting the technology

In essence, quantum cor
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overview of the fundame
some of the most freque



Goldman Sachs introc
and Development Eng
financial instruments a

Quantum algorithms could
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science behind the pricing o

JPMorgan Chase is one of the first financial institutions worldwide to invest in quantum computing and to build an internal team of scientists to work on new quantum algorithms and applications to address business use cases in finance, AI, optimization and cryptography.

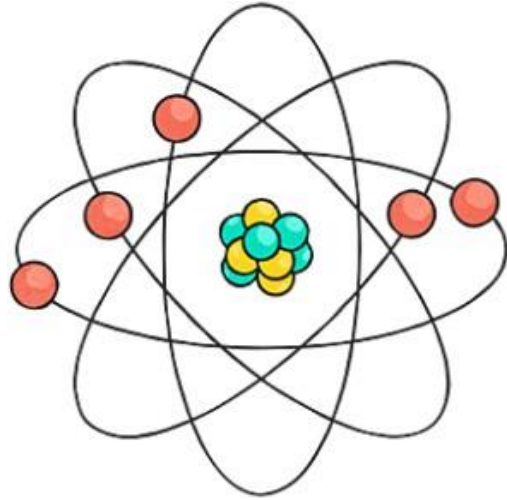
To date, the program has produced new quantum algorithms for use cases such as portfolio optimization, option pricing, risk analysis, and numerous applications in the realm of Machine Learning, ranging from fraud detection to Natural Language Processing.



Quantum Computing: The Key Concept

size

10^{-15} m



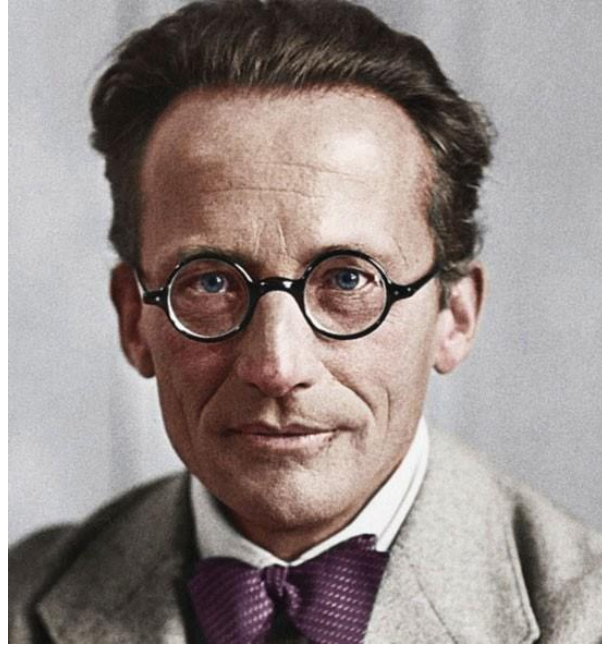
MICROSCOPIC
WORLD

10^{+6} m



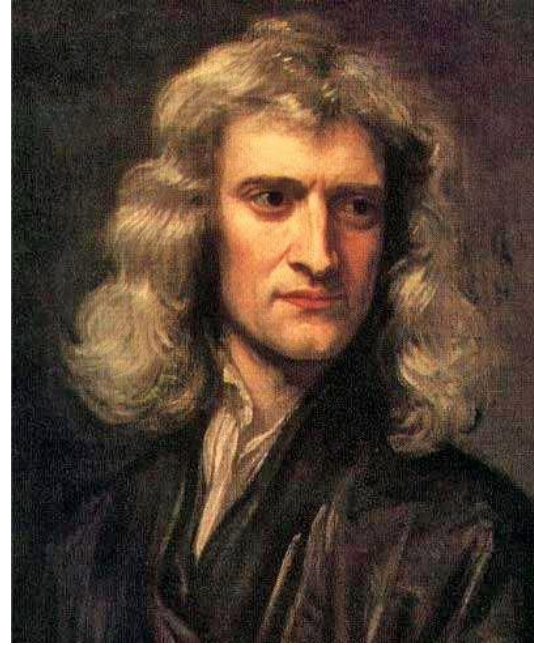
MACROSCOPIC
WORLD

Schroedinger



**MICROSCOPIC
WORLD**

Newton



**MACROSCOPIC
WORLD**

Schroedinger

$$i\hbar \frac{\partial}{\partial t} \Psi = \hat{H} \Psi$$

M**I**CROSCOPIC
WORLD

Newton

$$F = G \frac{m_1 m_2}{r^2}$$

M**A**CROSCOPIC
WORLD

The laws of physics are different depending on your scale (size)



The laws of physics are different depending on your scale (size)

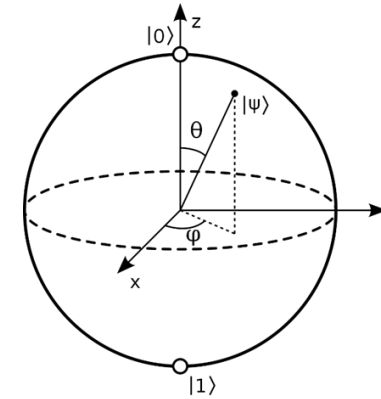
Quantum computers are machines that allow us to operate in the microscopic world – the quantum world



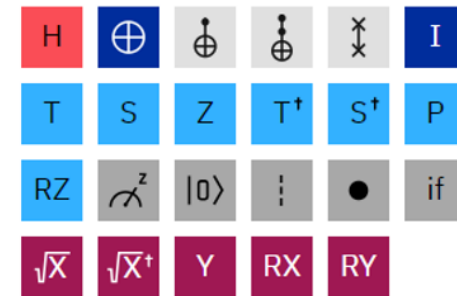
Quantum Computing: The Three Ingredients

The Three Ingredients

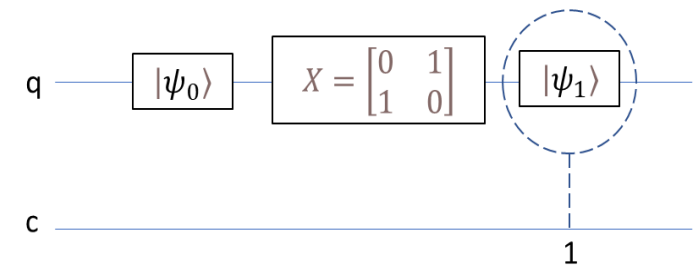
1: Qubits



2: Quantum Gates



3: Quantum Circuits



Ingredient 1: Qubits

Definition

qubit = quantum bit

Definition



bits = black/white



qubits = color

Definition



bits = black/white



qubits = color

Definition

A qubit $|\psi\rangle$ can be regarded as a vector in a two dimensional complex vector space, where $|0\rangle$ and $|1\rangle$ form its orthonormal basis, called the computational basis."

$$|\psi\rangle = a |0\rangle + b |1\rangle$$

The diagram illustrates the components of the qubit state equation $|\psi\rangle = a |0\rangle + b |1\rangle$. Red lines connect the terms to their descriptions: a diagonal line from $|\psi\rangle$ to 'qubit'; a diagonal line from a to 'complex, amplitude'; a vertical line from $|0\rangle$ to 'vector, direction $|0\rangle$ '; a diagonal line from b to 'complex, amplitude'; and a diagonal line from $|1\rangle$ to 'vector, direction $|1\rangle$ '.

qubit

complex,
amplitude

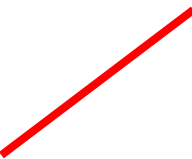
vector,
direction $|0\rangle$

complex,
amplitude

vector,
direction $|1\rangle$

Definition

A qubit $|\psi\rangle$ can be regarded as a vector in a two dimensional complex vector space, where $|0\rangle$ and $|1\rangle$ form its orthonormal basis, called the computational basis."

qubit  $|\psi\rangle =$

Definition

A qubit $|\psi\rangle$ can be regarded as a vector in a two dimensional complex vector space, where $|0\rangle$ and $|1\rangle$ form its orthonormal basis, called the computational basis."

qubit mixed state cat alive cat dead

$$|\psi\rangle = \frac{1}{\sqrt{2}} |\text{cat alive}\rangle + \frac{1}{\sqrt{2}} |\text{cat dead}\rangle$$
A diagram illustrating a qubit state as a superposition of two classical states. The equation $|\psi\rangle = \frac{1}{\sqrt{2}} |\text{cat alive}\rangle + \frac{1}{\sqrt{2}} |\text{cat dead}\rangle$ is shown. A red line points from the word 'qubit' to the ket symbol $|\psi\rangle$. The terms $|\text{cat alive}\rangle$ and $|\text{cat dead}\rangle$ are represented by a black silhouette of a sitting cat and a black silhouette of a dead cat, respectively, each enclosed in a red ket symbol \rangle . The coefficients $\frac{1}{\sqrt{2}}$ are written in red. The labels 'cat alive' and 'cat dead' are placed below their respective terms.

Definition

Definition

visual representation of a qubit

Definition**visual representation of a qubit**

Felix Bloch demonstrated that **any qubit can be represented as a point on the surface of a sphere** with two degrees of freedom (Euler's angles)

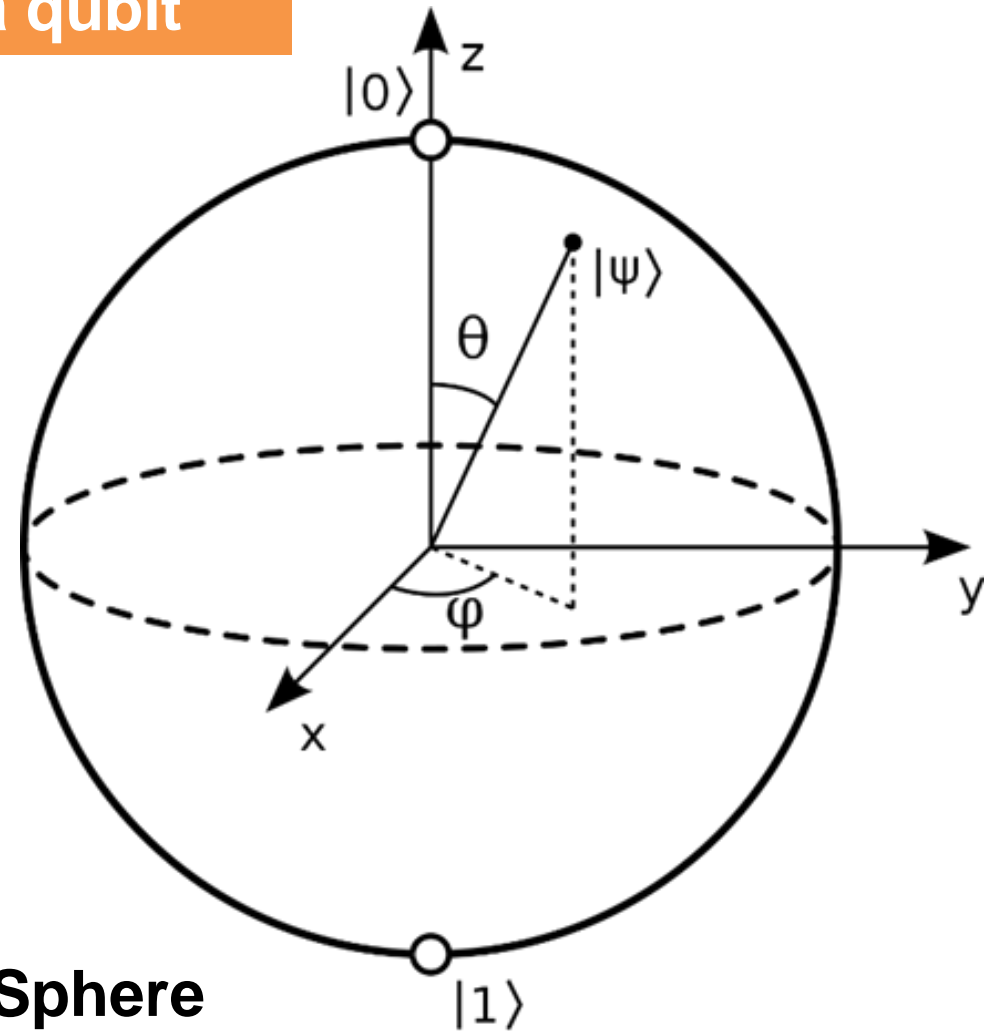
$$|\psi\rangle = \cos\left(\frac{\theta}{2}\right)|0\rangle + e^{i\varphi}\sin\left(\frac{\theta}{2}\right)|1\rangle$$

Definition

visual representation of a qubit

Felix Bloch demonstrated that **any qubit can be represented as a point on the surface of a sphere** with two degrees of freedom (Euler's angles)

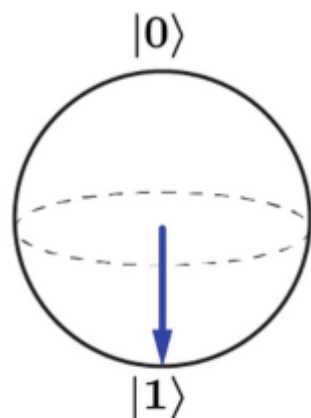
$$|\psi\rangle = \cos\left(\frac{\theta}{2}\right)|0\rangle + e^{i\varphi}\sin\left(\frac{\theta}{2}\right)|1\rangle$$



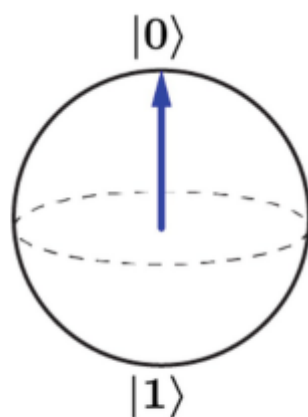
Bloch Sphere

Definition

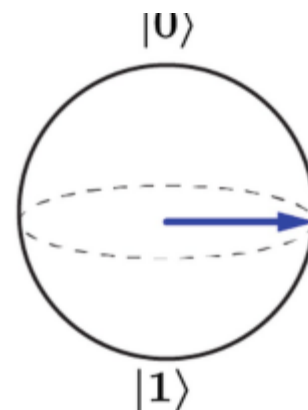
visual representation of a qubit



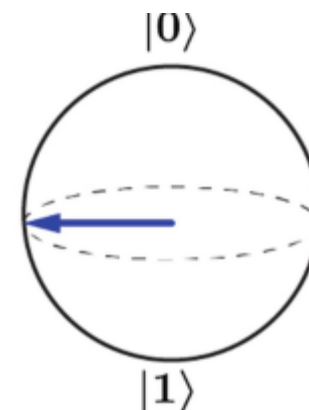
$$|\Psi\rangle = |1\rangle$$



$$|\Psi\rangle = |0\rangle$$



$$|\Psi\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$$



$$|\Psi\rangle = \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle)$$

Examples

Ingredient 2: Quantum Gates

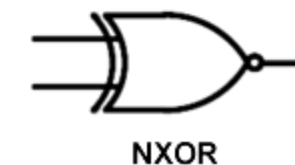
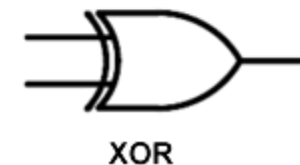
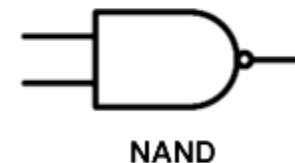
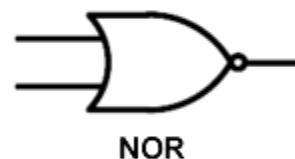
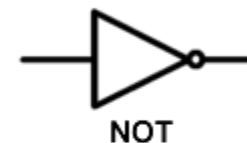
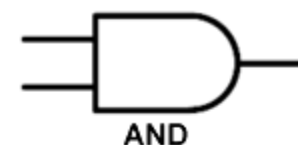
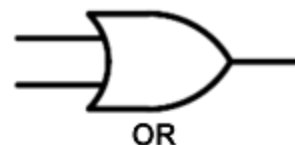
Quantum Gates

Classical Gates

Classical gates allow the transformation of certain inputs into certain outputs according to some logical rules.

Inputs: binary (0, 1)

Output: binary (0, 1)



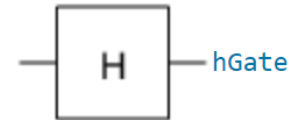

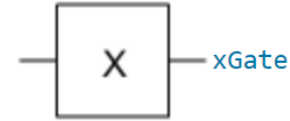

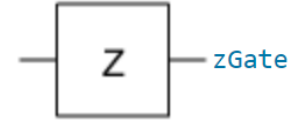
Quantum Gates

Quantum Gates

Quantum gates allow the transformation of quantum information into quantum information following certain matrix rules.

Inputs: qubit

Output: qubit

Creation Function	Gate Name	Matrix Representation
	Hadamard gate	$\frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$
	Identity gate	$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$
	Pauli X gate	$\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$
	Pauli Y gate	$\begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}$
	Pauli Z gate	$\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$

Ingredient 3: Quantum Circuit

Quantum Circuit

a series of quantum gates (matrix operations) applied to qubits

Quantum Circuit

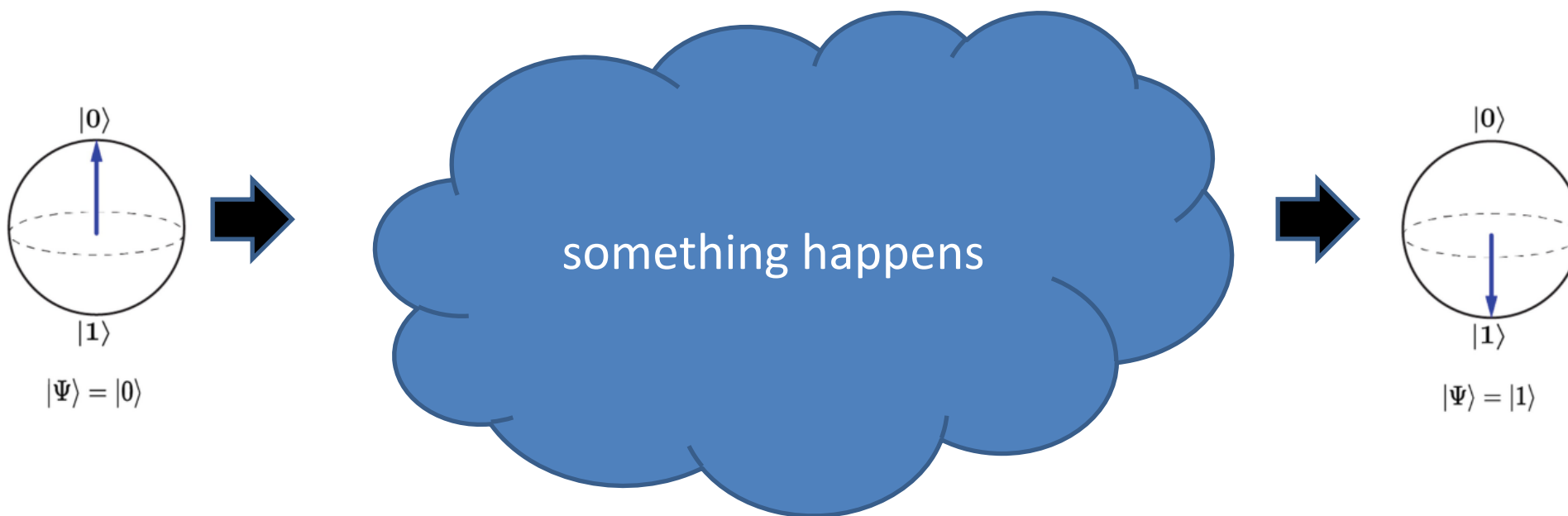
a series of quantum gates (matrix operations) applied to qubits



Quantum Circuit

Example: circuit to flip qubit (i.e. Hello World)

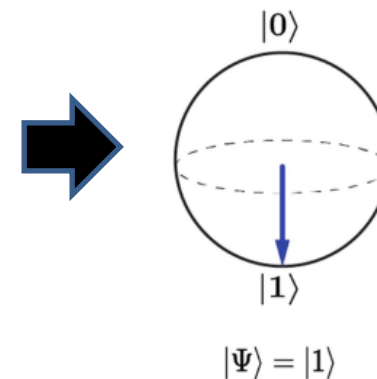
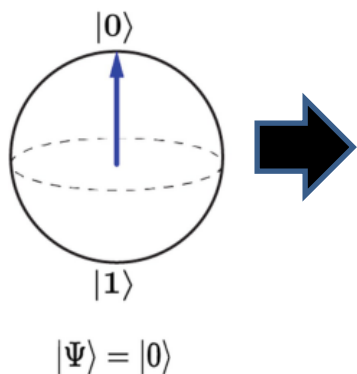
a series of quantum gates (matrix operations) applied to qubits



Quantum Circuit

Example: circuit to flip qubit (i.e. Hello World)

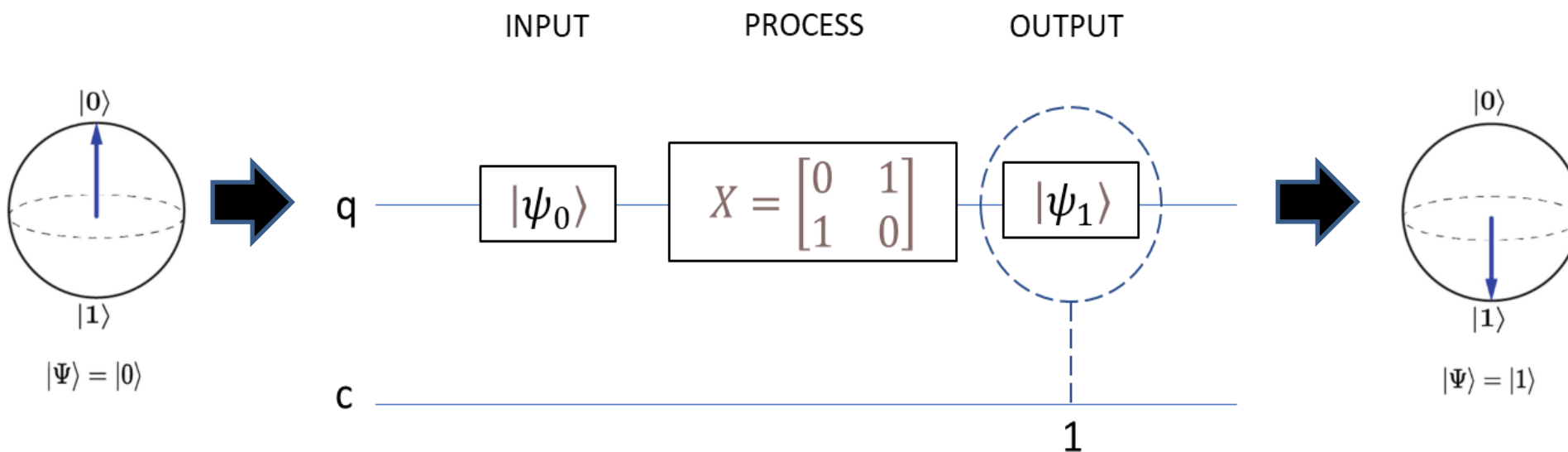
a series of quantum gates (matrix operations) applied to qubits



Quantum Circuit

Example: circuit to flip qubit (i.e. Hello World)

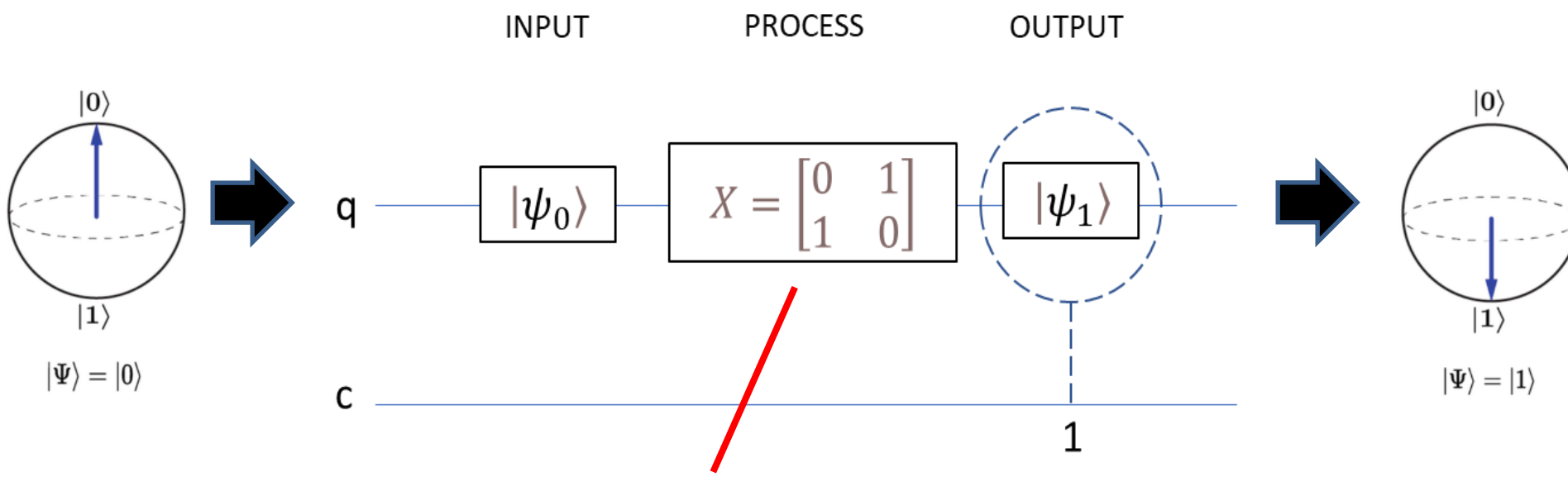
a series of quantum gates (matrix operations) applied to qubits



Quantum Circuit

Example: circuit to flip qubit (i.e. Hello World)

a series of quantum gates (matrix operations) applied to qubits



quantum X gate (NOT gate)

Now you try !

Quantum Computing: **In Practice**

Now you try !

Now you try !

www.ibm.com/quantum

Alonso Pena

IBM Quantum Platform

Recent jobs

0

Pending

35

Completed jobs

Job ID	Status
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cjj07bj3smr2evnriss0	Completed
cjj065r3smr2evnrhusg	Completed
chnqu40uhh1ldcoe6e50	Completed
cgvuc0ngbjvrfp7ko2mg	Completed

Your systems



8

Documentation

Search docs

Qiskit Runtime

Introduction to primitives

Dynamic Circuits

New

Simulators



IBM Quantum



Now you try !

Welcome, Alonso Pena



Graphically build circuits with
IBM Quantum Composer

[Launch Composer](#)

Develop quantum experiments in
IBM Quantum Lab

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Register FREE and use the online platform

Untitled circuit *Saved*

File

Edit

View

Visualizations seed

582

Setup and run



Operations



Left alignment



Inspect



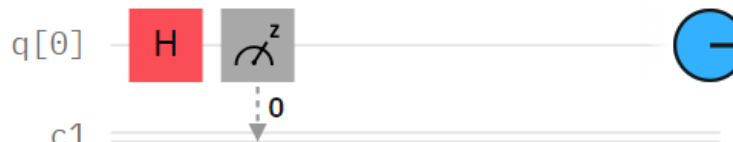
Qiskit



Search



H	\oplus	\oplus	\oplus	\otimes	I
T	S	Z	T^\dagger	S^\dagger	P
RZ	\curvearrowright^z	$ 0\rangle$	$ 1\rangle$	\bullet	if
\sqrt{X}	\sqrt{X}^\dagger	Y	RX	RY	RXX
RZZ	U	RCCX	RC3X		

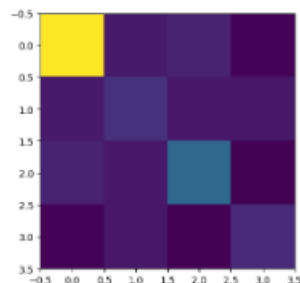
[Open in Quantum Lab](#)

```
1 from qiskit import
   QuantumRegister,
   ClassicalRegister,
   QuantumCircuit
2 from numpy import pi
3
4 qreg_q = QuantumRegister(1,
   'q')
5 creg_c = ClassicalRegister(1,
   'c')
6 circuit = QuantumCircuit
   (qreg_q, creg_c)
7
8 circuit.h(qreg_q[0])
9 circuit.measure(qreg_q[0],
   creg_c[0])
```

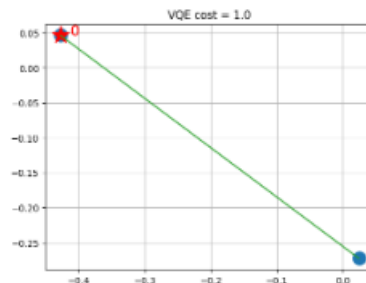
Now you try !

Construct your own quantum circuits at **IBM Quantum Composer**

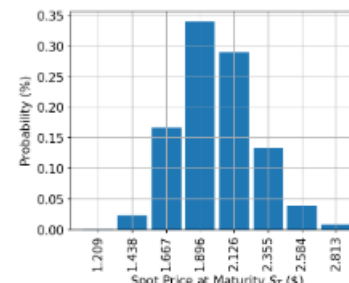
Finance Tutorials 🏹

Quantum Amplitude
Estimation

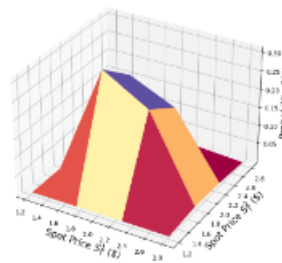
Portfolio Optimization



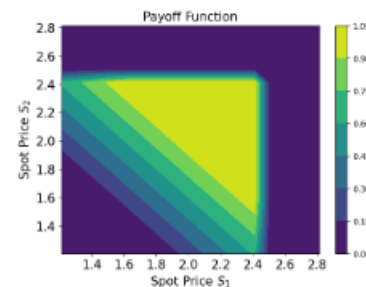
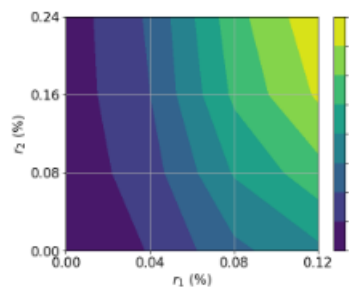
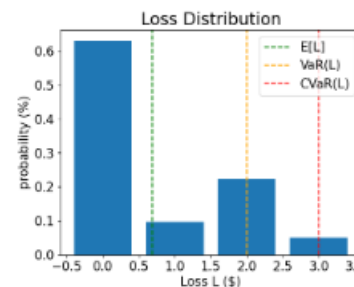
Portfolio Diversification

Pricing European Call
OptionsPricing European Put
Options

Pricing Bull Spreads



Pricing Basket Options

Pricing Asian Barrier
SpreadsPricing Fixed-Income
Assets

Credit Risk Analysis

Now you try !

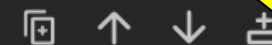
Write and run your own quantum Python programs at **IBM Quantum Lab**



Now you try !



▾ Pricing European Call Options



Introduction

Suppose a **European call option** with strike price K and an underlying asset whose spot price at maturity S_T follows a given random distribution. The corresponding payoff function is defined as:

$$\max\{S_T - K, 0\}$$

In the following, a quantum algorithm based on amplitude estimation is used to estimate the expected payoff, i.e., the fair price before discounting, for the option:

Write and run your own quantum Python programs at **IBM Quantum Lab**



IBM Quantum Learning

FileEditViewRunKernelTabsSettingsHelp

01_portfolio_optimization.ip

python

Now you try !

Portfolio Optimization

Introduction

This tutorial shows how to solve the following mean-variance portfolio optimization problem for n assets:

$$\min_{x \in \{0,1\}^n} q x^T \Sigma x - \mu^T x$$

subject to: $1^T x = B$

where we use the following notation:

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Now you try !

▼ Credit Risk Analysis

Introduction

This tutorial shows how quantum algorithms can be used for credit risk analysis. More precisely, how Quantum Amplitude Estimation (QAE) can be used to estimate risk measures with a quadratic speed-up over classical Monte Carlo simulation. The tutorial is based on the following papers:

- [Quantum Risk Analysis](#). Stefan Woerner, Daniel J. Egger. [Woerner2019]
- [Credit Risk Analysis using Quantum Computers](#). Egger et al. (2019) [Egger2019]

A general introduction to QAE can be found in the following paper:

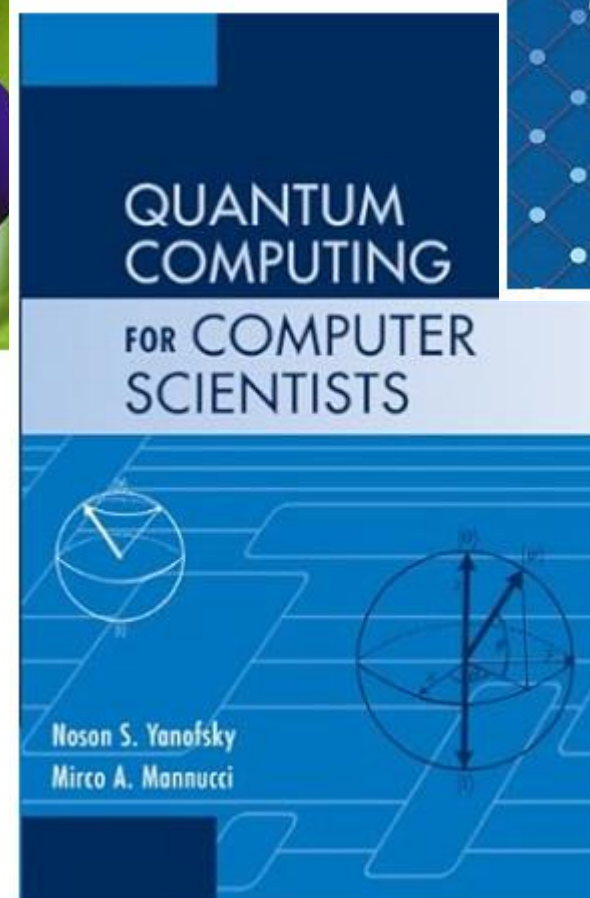
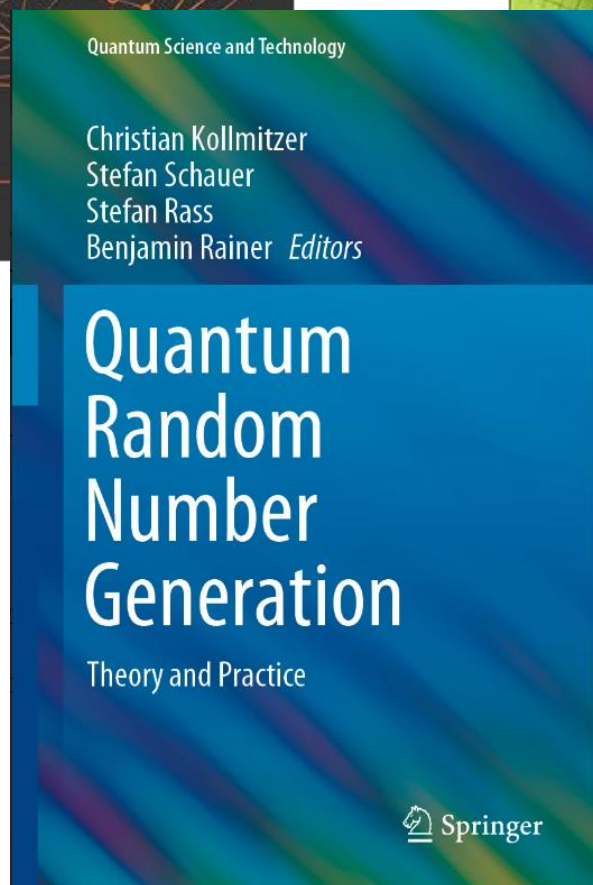
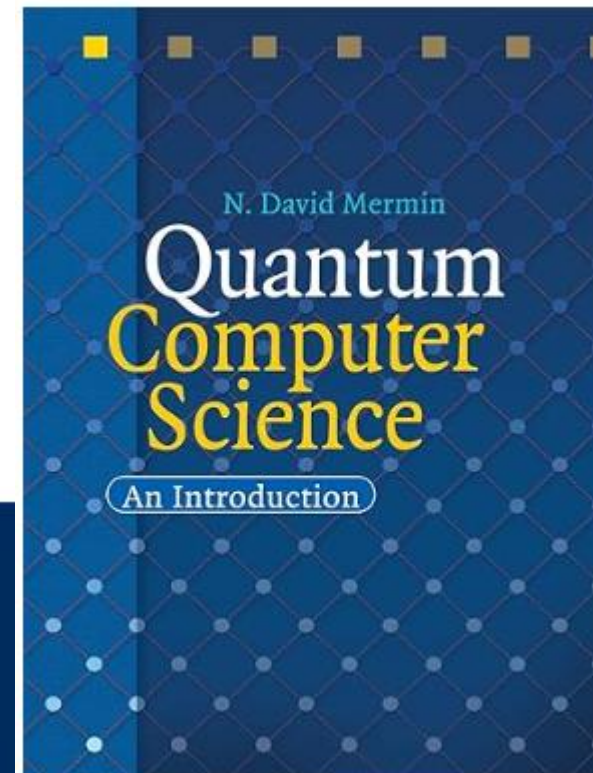
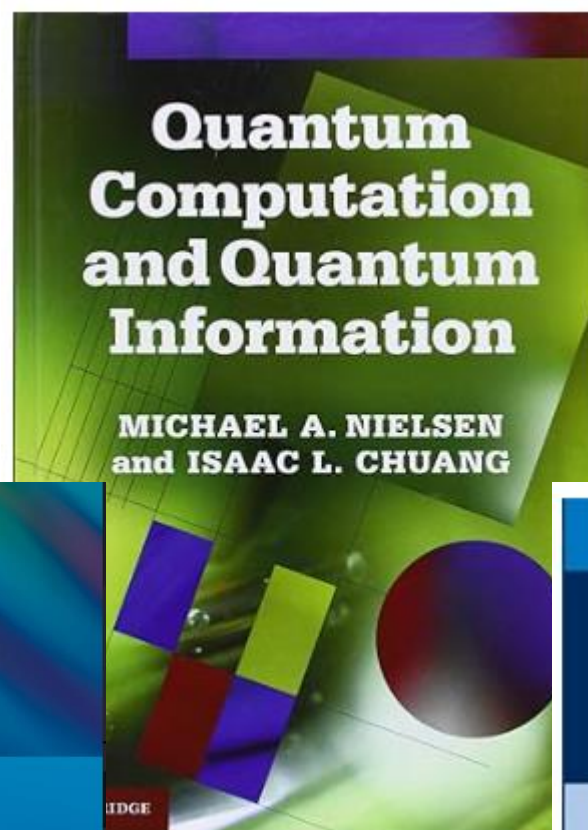
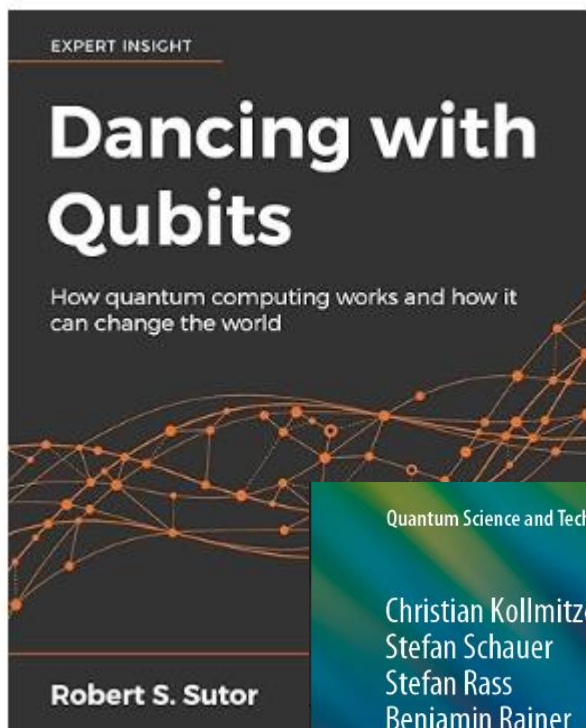
Write and run your own quantum Python programs at **IBM Quantum Lab**





Thank you

Quantum Computing: References





Quantum computing for financial risk measurement

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Abstract

Quantum computing allows a significant speed-up over traditional CPU- and GPU-based algorithms when applied to particular mathematical challenges such as optimisation and simulation. Despite promising advances and extensive research in hard- and software developments, currently available quantum systems are still largely limited in their capability. In line with this, practical applications in quantitative finance are still in their infancy. This paper analyses requirements and concrete approaches for the application to risk management in a financial institution. On the examples of Value-at-Risk for market risk and Potential Future Exposure for counterparty credit risk, the main contribution lies in going beyond textbook illustrations and instead



OPEN Copula-based risk aggregation with trapped ion quantum computers

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Copulas are mathematical tools for modeling joint probability distributions. In the past 60 years they have become an essential analysis tool on classical computers in various fields. The recent finding that copulas can be expressed as maximally entangled quantum states has revealed a promising approach to practical quantum advantages: performing tasks faster, requiring less memory, or, as we show, yielding better predictions. Studying the scalability of this quantum approach as both the precision and the number of modeled variables increase is crucial for its adoption in real-world applications. In this paper, we successfully apply a Quantum Circuit Born Machine (QCBM) based approach to modeling 3- and 4-variable copulas on trapped ion quantum computers. We study the training of QCBMs with different levels of precision and circuit design on a simulator and a state-of-the-art trapped ion quantum computer. We observe decreased training efficacy due to the increased complexity in parameter optimization as the models scale up. To address this challenge, we introduce an annealing-inspired strategy that dramatically improves the training results. In our end-to-end tests, various configurations of the quantum models make a comparable or better prediction in risk aggregation tasks than the standard classical models.

Option Pricing using Quantum Computers

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We present a methodology to price options and portfolios of options on a gate-based quantum computer using amplitude estimation, an algorithm which provides a quadratic speedup compared to classical Monte Carlo methods. The options that we cover include vanilla options, multi-asset options and path-dependent options such as barrier options. We put an emphasis on the implementation of the quantum circuits required to build the input states and operators needed by amplitude estimation to price the different option types. Additionally, we show simulation results to highlight how the circuits that we implement price the different option contracts. Finally, we examine the performance of option pricing circuits on quantum hardware using the IBM Q Tokyo quantum device. We employ a simple, yet ef-

ist for the simplest types of options [2], the simplifying assumptions on the market dynamics required for the models to provide closed-form solutions often limit their applicability [3]. Hence, more often than not, numerical methods have to be employed for option pricing, with Monte Carlo being one of the most popular due to its flexibility and ability to generically handle stochastic parameters [4, 5]. However, despite their attractive features in option pricing, classical Monte Carlo methods generally require extensive computational resources to provide accurate option price estimates, particularly for complex options. Because of the widespread use of options in the finance industry, accelerating their convergence can have a significant impact in the operations of a financial institution.

By leveraging the laws of quantum mechanics a quantum computer [6] may provide novel ways to

ARTICLE OPEN

Quantum risk analysis

Stefan Woerner ¹ and Daniel J. Egger¹

We present a quantum algorithm that analyzes risk more efficiently than Monte Carlo simulations traditionally used on classical computers. We employ quantum amplitude estimation to price securities and evaluate risk measures such as Value at Risk and Conditional Value at Risk on a gate-based quantum computer. Additionally, we show how to implement this algorithm and how to trade-off the convergence rate of the algorithm and the circuit depth. The shortest possible circuit depth—growing polynomially in the number of qubits representing the uncertainty—leads to a convergence rate of $O(M^{-2/3})$, where M is the number of samples. This is already faster than classical Monte Carlo simulations which converge at a rate of $O(M^{-1/2})$. If we allow the circuit depth to grow faster, but still polynomially, the convergence rate quickly approaches the optimum of $O(M^{-1})$. Thus, for slowly increasing circuit depths our algorithm provides a near quadratic speed-up compared to Monte Carlo methods. We demonstrate our algorithm using two toy models. In the first model we use real hardware, such as the IBM Q Experience, to price a Treasury-bill (T-bill) faced by a possible interest rate increase. In the second model, we simulate our algorithm to illustrate how a quantum computer can determine financial risk for a two-asset portfolio made up of government debt with different maturity dates. Both models confirm the improved convergence rate over Monte Carlo methods. Using simulations, we also evaluate the impact of cross-talk and energy relaxation errors.

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INTRODUCTION

Risk management plays a central role in the financial system. Value at risk (VaR),¹ a quantile of the loss distribution, is a widely

It has already been shown how AE can be used to price financial derivatives with the Black–Scholes model.^{13,14}

In this article, we extend the use of AE to the calculation of

Quantum Computation for Pricing the Collateralized Debt Obligations

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Collateralized debt obligation (CDO) has been one of the most commonly used structured financial products and is intensively studied in quantitative finance. By setting the asset pool into different tranches, it effectively works out and redistributes credit risks and returns to meet the risk preferences for different tranche investors. The copula models of various kinds are normally used for pricing CDOs, and the Monte Carlo simulations are required to get their numerical solution. Here we implement two typical CDO models, the single-factor Gaussian copula model and Normal Inverse Gaussian copula model, and by applying the conditional independence approach, we manage to load each model of distribution in quantum circuits. We then apply quantum amplitude estimation as an alternative to Monte Carlo simulation for CDO pricing. We demonstrate the quantum computation results using IBM Qiskit. Our

only a few stochastic equations for derivative pricing have found analytical solutions^{[20][21]}, while most can only be solved numerically by repeating random settings a great many times in an uncertainty distribution (*e.g.* normal or log-normal distribution), which therefore consumes much time. The quantum amplitude estimation (QAE) algorithm was raised^[32] in 2002. It is newly suggested as a promising alternative to the Monte Carlo method, as it shows a quadratic speedup comparing to the latter^[10]. So far, applications of QAE for option pricing^[11] and credit risk analysis^[12] have been demonstrated.

Considering the wide use of Monte Carlo simulation and the large variety of pricing models, the involvement of quantum techniques in finance is still at its infancy. Credit derivatives are frequently mentioned financial instruments because of the strong demand for tackling default risks in finance industry. Collateralized debt obligation (CDO) is a multi-name credit derivative backed on a pool of portfolios of defaultable assets (loans, bonds,

Quantum Algorithms for Portfolio Optimization

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ABSTRACT

We develop the first quantum algorithm for the constrained portfolio optimization problem. The algorithm has running time

$\tilde{O}\left(n\sqrt{r}\frac{\zeta\kappa}{\delta^2}\log(1/\epsilon)\right)$, where r is the number of positivity and budget constraints, n is the number of assets in the portfolio, ϵ the desired precision, and δ, κ, ζ are problem-dependent parameters related to the well-conditioning of the intermediate solutions. If only a moderately accurate solution is required, our quantum algorithm can achieve a polynomial speedup over the best classical algorithms with complexity $\tilde{O}\left(\sqrt{r}n^\omega\log(1/\epsilon)\right)$, where ω is the matrix multiplication exponent that has a theoretical value of around 2.373, but is closer to 3 in practice.

We also provide some experiments to bound the problem-dependent factors arising in the running time of the quantum algorithm, and these experiments suggest that for most instances the quantum algorithm can potentially achieve an $O(n)$ speedup over its classical counterpart.

KEYWORDS

It has been suggested that quantum techniques like Feynman integrals could be useful for option pricing [3]. There has also been experimental work where the IBM quantum computers have been used to explore quadratic speedups for option pricing [24, 29] and work on quadratic speedups for option pricing using Monte Carlo methods [26]. While some of these results lack provable guarantees, they indicate the strong interest in both the quantum algorithms and mathematical Finance communities in developing applications of quantum computers to computational finance.

Very recently, Lloyd and Rebentrost [27] proposed a quantum algorithm for the unconstrained portfolio optimization problem. Their algorithm uses quantum linear system solvers to obtain speedups for portfolio optimization problems that can be reduced to unconstrained quadratic programs, which in turn are reducible to a single linear system. The main limitation of their algorithm is that it can not incorporate positivity or budget constraints, thus restricting its applicability to real world problems that can have complex budget constraints. The reason for this limitation is algorithmic, the constrained portfolio optimization problem is known to be equivalent to quadratic programming (QP), a class of optimization problems

Credit Risk Analysis using Quantum Computers

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(Dated: July 9, 2019)

We present and analyze a quantum algorithm to estimate credit risk more efficiently than Monte Carlo simulations can do on classical computers. More precisely, we estimate the economic capital requirement, i.e. the difference between the Value at Risk and the expected value of a given loss distribution. The economic capital requirement is an important risk metric because it summarizes the amount of capital required to remain solvent at a given confidence level. We implement this problem for a realistic loss distribution and analyze its scaling to a realistic problem size. In particular, we provide estimates of the total number of required qubits, the expected circuit depth, and how this translates into an expected runtime under reasonable assumptions on future fault-tolerant quantum hardware.

I. INTRODUCTION

Economic Capital, a key tool of risk management, is computed by financial service firms to determine the amount of risk capital that they require to remain solvent in the face of adverse yet realistic conditions [1]. Financial service firms are exposed to many forms of risk [2] such as credit risk which is the risk of a monetary loss resulting from a counterparty failing to meet a financial obligation [3, 4]. For instance, a payment may not be made in due time or at all. Risk metrics such as Value at Risk and the Economic Capital Requirement (ECR)

algorithms on a gate based quantum computer. In Section IV, we show simulation results for small instances of the considered models. Section V analyzes the scaling of the algorithm for problems of realistic size as well as the resulting quantum advantage.

II. CREDIT RISK ANALYSIS

ECR summarizes in a single figure the amount of capital (or own funds) required to remain solvent at a given confidence level (usually linked to the risk appetite or

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